Department of Mathematics, Howard University
Final Examination
College Algebra I: Math 006, Fall 2009
Each question in Part A carries 5 points.Each full question in Part B carries 20 points.
Do not use any calculator. Time: 2 hours.

PART A

1. Simplify: $\dfrac{\sqrt{8}x^{\frac{3}{2}}}{\sqrt{2}x^{\frac{2}{3}}}$

2. Ratioanlize: $\dfrac{1}{(3-\sqrt{2})(\sqrt{8}-3)}$

3. Is (x+2) a factor of x^3-8?

4. Solve the equation : $\sqrt{(x^2-x-1)} = x+2$

5. Solve the inequality : $|x+2| \leq 5$

6. Solve the equation: $\dfrac{x+3}{x+4} = \dfrac{x+1}{x+2}$

7. Solve the equation $x^2+4x+1=0$ by completing to a square.

8. Find the maximum number of negative zeroes of the polynomial
   $P(x) = x^4+x^3+x^2+x-1$

9. Find the remainder when $x^4+x^3+x^2+x-1$ is divided by x-2

10. Write an equation to the line joining the points P(2,3) and Q(-4,7)

11. Write an equation to the line perpendicular to the line with equation 3x+4y =1 and passing thro the point (4,1).

12. Find the center and radius of the circle whose equation is
    $x^2+y^2-2x+4y-1=0$

13. Which of the following relations represents a function?
    (a) (-3,9),(-2,4),(-3,5),(1,1)
    (b) (1,3),(4,-2),(-3,5), (1,7)
    (c) (2,5),(4,6), (6,7), (8,8)

14. f(x) =2 x^3+mx^2+4x-5 and f(2)=3, what is the value of m?

15. If $f(x) = x^2+1$, find f(f(x) and f(x).f(x).

16. If $f(x) = 1 - x^2$, find $\dfrac{f(x+h)-f(x)}{h}$.

PART B

I A boat maintained a constant speed of five miles per hour relative to the water. It went 8 miles upstream and immediately returned. The total time elapsed was 5 hours. Determine the speed of the current.

II Draw the graphs of the following functions in the same set of coordinate axes:

(a) $y = x$

(b) $y = \dfrac{1}{x}$

(c) $y = \dfrac{1}{x^2}$

(d) $y = x + \dfrac{x+1}{x^2}$, $x>0$.

III. Let $f(x) = \dfrac{2x}{x-2}$.

(a) What are the domain and range of f(x)?

(b) What are the x-intercepts, if any and the y-intercepts, if any?

(c) Draw a neat graph of f(x).

IV.

$$f(x) = \begin{cases} x^4 & \text{if } x \le 0 \\ x & \text{if } 0 < x \le 4 \end{cases}$$

(a)Sketch the graph of f(x)

(b)Find where the horizontal line y=16 intersects the graph of f(x).

(c)Find the interval on which f(x) is increasing, decreasing or neither.

V. Solve the equation $x^4 - 4x^2 + 3 = 0$

(a) by converting the equation to a quadratic equation.

(b) by applying the Descates'Rule of Signs, find the maximum possible number of real roots, list all the possible rational roots, and check,by Synthetic Division, which of the possible rational roots are actual roots. Then find the other roots.

VI. A manufacturer can produce blank videotape cassettes at a cost of \$2 apiece. The cassettes have been selling for \$5 apiece and at that rate 4,000 cassettes are sold each month. The manufacturer is planning to raise the selling price, and he estimates that for each \$1 increase ,500 fewer cassettes will be sold each month. What should be the new selling price per cassette to maximize the monthly profit? (Hint: Let the new selling price per cassette be x dollars. Express the profit as a quadratic function of x, complete to a square and draw the graph).

Alg I Fall 2009 Final Exam Solutions

#1 $\dfrac{\sqrt{8}\,x^{3/2}}{\sqrt{2}\,x^{2/3}} = \left(\dfrac{\sqrt{8}}{\sqrt{2}}\right)\cdot\left(\dfrac{x^{3/2}}{x^{2/3}}\right) = \left(\sqrt{\dfrac{8}{2}}\right)\left(x^{3/2-2/3}\right)$

$= (\sqrt{4})\left(x^{9/6-4/6}\right) = 2x^{5/6}$

#2 $\dfrac{1}{(3-\sqrt{2})(\sqrt{8}-3)} = \dfrac{(3+\sqrt{2})(\sqrt{8}+3)}{[(3-\sqrt{2})(3+\sqrt{2})][(\sqrt{8}-3)(\sqrt{8}+3)]}$

$= \dfrac{3\sqrt{8}+9+4+3\sqrt{2}}{(9-2)(8-9)} = \dfrac{6\sqrt{2}+13+3\sqrt{2}}{(7)(-1)}$

$= -\left(\dfrac{9\sqrt{2}+13}{7}\right)$

#3 No because:

Reason #1:

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & -8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & -16 \end{array}$$

remainder is not zero

or

Reason #2: $x^3-8 = (x-2)(x^2+2x+4)$

AND x^2+2x+4 does not factor because it has negative discriminant $(b^2-4ac<0)$

#4 $\sqrt{x^2-x-1} = x+2$

$x^2-x-1 = (x+2)^2$

$x^2-x-1 = x^2+4x+4$

$5x+5=0$

$5x=-5$

$x=-1$

Check:

RHS: $-1+2=1$

LHS: $\sqrt{(-1)^2-(-1)-1}$

$= \sqrt{1+1-1}$

$= \sqrt{1}$

$= 1$

checks.

#5 Solve $|x+2| \le 5$

$x+2 \le 5$

$x \le 3$

$-(x+2) \le 5$

$x+2 \ge -5$

$x \ge -7$

$-7 \le x \le 3$

#6 Solve $\dfrac{x+3}{x+4} = \dfrac{x+1}{x+2}$

$(x+3)(x+2) = (x+1)(x+4)$

$x^2+5x+6 = x^2+5x+4$

$6 = 4$ contradiction

No Solution

#7 $x^2+4x+1=0$

$x^2+4x=-1$

$x^2+4x+4=-1+4$

$(x+2)^2=3$

$\sqrt{(x+2)^2}=\sqrt{3}$

$|x+2|=\sqrt{3}$ $\quad (x+2=\sqrt{3}$ or $-(x+2)=\sqrt{3})$

$x+2=\pm\sqrt{3}$

$x=-2\pm\sqrt{3}$

#8 $P(x)=x^4+x^3+x^2+x-1$

$P(-x)=\underbrace{x^4-x^3}_{1}\underbrace{+x^2}_{2}\underbrace{-x}_{3}-1$ $\quad$ 3 changes of sign

$\therefore$ P has at most 3 negative zeros

#9

2	1	1	1	1	−1
		2	6	14	30
	1	3	7	15	29

or

$$\begin{array}{r} x^3+3x^2+7x+15 \\ x-2\,\overline{)\,x^4+x^3+x^2+x-1} \\ -(x^4-2x^3) \\ 3x^3+x^2 \\ -(3x^3-6x^2) \\ 7x^2+x \\ -(7x^2-14x) \\ 15x-1 \\ -(15x-30) \\ 29 \end{array}$$

#10 $P(2,3),\ Q(-4,7)$

$m=\frac{\Delta y}{\Delta x}=\frac{3-7}{2+4}=\frac{-4}{6}=\frac{-2}{3}$

$y-y_1=m(x-x_1)\Rightarrow y-3=-\frac{2}{3}(x-2)$ (pt-slope form)

$y-3=-\frac{2}{3}x+\frac{4}{3}$

$y=-\frac{2}{3}x+\frac{13}{3}$ (slope-intercept form)

#11 $3x+4y=1$

$4y=-3x+1$

$y=-\frac{3}{4}x+\frac{1}{4}$ $\quad$ So the slope of the original line is $-\frac{3}{4}$

Therefore the slope of a perpendicular line is $+\frac{4}{3}$

So we want the line with slope $4/3$ through the point $(4,1)$

pt-slope form is easiest: $y-1=\frac{4}{3}(x-4)$

pt-intercept is most standard: $y-1=\frac{4}{3}x-\frac{16}{3}$

$y=\frac{4}{3}x+1-\frac{16}{3}$

$y=\frac{4}{3}x-\frac{13}{3}$

#12 $x^2+y^2-2x+4y-1=0$

$(x^2-2x)+(y^2+4y)=1$

$(x^2-2x+1)+(y^2+4y+4)=1+1+4$

$(x-1)^2+(y+2)^2=6$

center : $(1,-2)$
radius : $\sqrt{6}$

#13 a) No, because $(-3,9)$ and $(-3,5)$ fail vertical line test

b) No, because $(1,3)$ and $(1,7)$ " " " "

c) Yes

#14 $f(x)=2x^3+mx^2+4x-5$

$3=f(2)=2(2)^3+m(2)^2+4(2)-5$

$3=2(8)+m(4)+8-5$

$3=16+4m+3$

$-16=4m$

$m=-4$

#15 $f(x)=x^2+1$

$f(f(x))=f(x^2+1)=(x^2+1)^2+1=x^4+2x^2+2$

$f(x)\cdot f(x)=f^2(x)=(x^2+1)^2=x^4+2x^2+1$

#16 $f(x)=1-x^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{\overbrace{1-(x+h)^2}^{f(x+h)}-\overbrace{(1-x^2)}^{f(x)}}{h}$$

$$=\frac{1-(x^2+2xh+h^2)-1+x^2}{h}$$

$$=\frac{1-x^2-2xh-h^2-1+x^2}{h}$$

$$=\frac{-2xh-h^2}{h}$$

$$=\frac{-h(+2x+h)}{h}=-(2x+h)$$

I. dist = (speed) × (time)

	dist.	speed	time
upstream	8	$5-x$	t_1
downstream	8	$5+x$	t_2

where $t_1 + t_2 = 5$ and $t_1 = \frac{8}{5-x}$, $t_2 = \frac{8}{5+x}$

and x = speed of water

$$5 = t_1 + t_2 = \frac{8}{5-x} + \frac{8}{5+x}$$

$$\frac{5}{8} = \frac{1}{5-x} + \frac{1}{5+x}$$

$$= \frac{5+x}{(5+x)(5-x)} + \frac{5-x}{(5-x)(5+x)}$$

$$= \frac{5+x+5-x}{(5+x)(5-x)}$$

$$= \frac{10}{(5+x)(5-x)}$$

$$\frac{1}{16} = \frac{1}{(5+x)(5-x)}$$

$$16 = (5+x)(5-x)$$

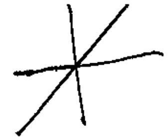
$$= 25 - x^2$$

$$x^2 = 25 - 16 = 9$$

$x = \pm 3$. The problem is set up so that the speed x is positive.

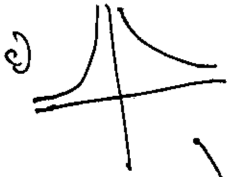
The speed of the water is 3 mph.

II. a)



b)

c)



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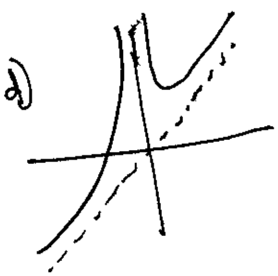


vertical asymp is $x=0$

Oblique asymp is $y=x$.

$x + \frac{x+1}{x^2} = \frac{x^3+x+1}{x^2}$

Using Decartes Rule of signs x^3+x+1 has no positive zeros, and must have one negative zero. This is enough for sketch graph.

III. $f(x) = \frac{2x}{x-2}$

a) $\mathrm{dom}(f) = \{x \mid x \neq 2\}$

$\mathrm{ran}(f) = \{x \mid x \neq 2\}$ ← (Deduce this from Part c)

b) x-intercepts – set $y=0$: $0 = \frac{2x}{x-2} \Rightarrow x=0$

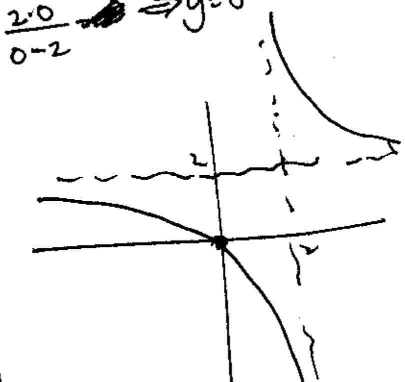
y-intercept – set $x=0$: $y = \frac{2 \cdot 0}{0-2} \Rightarrow y=0$

c) vertical asymptote : $x=2$

horizontal asymptope $y=2$

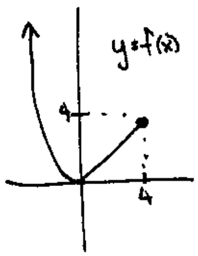
$$y = \frac{2x}{x-2} = 2 + \frac{4}{x-2}$$

(Shift $y = \frac{4}{x}$ two units up and two units to right)



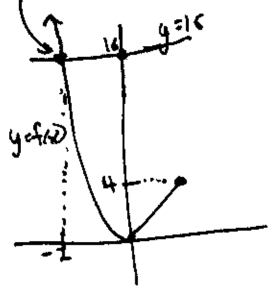
IV. $f(x) = \begin{cases} x^4 & x \le 0 \\ x, & 0 < x \le 4 \end{cases}$

ⓐ



ⓑ

Intersection of $y=f(x)$ and $y=16$



From the graph we see that the horizontal line $y=16$ only intersects the graph at the point $(-2,16)$

ⓒ Increasing: $(0,4) = \{x \mid 0 < x < 4\}$

Decreasing: $(-\infty, 0) = \{x \mid x < 0\}$

V. ⓐ $x^4 - 4x^2 + 3 = 0$. Let $u = x^2$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$(x^2-3)(x^2-1) = 0$

$(x^2-3)(x-1)(x+1) = 0$

$x^2 - 3 = 0$ $x - 1 = 0$ $x + 1 = 0$

$x^2 = 3$ $x = 1$ $x = -1$

$|x| = \sqrt{3}$

$x = \pm\sqrt{3}$

Solution set is

$\{\sqrt{3}, -\sqrt{3}, 1, -1\}$

V. ⓑ $x^4 - 4x^2 + 3 = 0$

2 changes in sign $\Rightarrow$ 2 possible positive roots.

$(-x)^4 - 4(-x)^2 + 3 = x^4 - 4x^2 + 3 = 0$

Again, 2 possible negative roots

The only rational possibilities are the numbers that can be formed using ± 1 or ± 3 in the numerator and ± 1 in denominator — i.e. rational possibilities are $\pm 1, \pm 3$

Check:

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -4 & 0 & 3 \\ & & 3 & 9 & 15 & 45 \\ \hline & 1 & 3 & 5 & 15 & 48 \end{array} \leftarrow \text{NO}$$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -4 & 0 & 3 \\ & & -3 & 9 & -15 & 45 \\ \hline & 1 & -3 & 5 & -15 & 48 \end{array} \leftarrow \text{NO}$$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -4 & 0 & 3 \\ & & 1 & 1 & -3 & -3 \\ \hline & 1 & 1 & -3 & -3 & 0 \end{array} \leftarrow \text{YES}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -4 & 0 & 3 \\ & & -1 & 1 & 3 & -3 \\ \hline & 1 & -1 & -3 & 3 & 0 \end{array} \leftarrow \text{YES}$$

(V continued)

Since $(x-1)$ is a factor, using the division above

$$x^4-4x^2+3=(x-1)(x^3+x^2-3x-3)$$

Since $(x+1)$ is a factor, it must divide x^3+x^2-3x-3:

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -3 & -3 \\ & & -1 & 0 & 3 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

So $x^4-4x^2+3=(x-1)(x+1)(x^2-3)$
$=(x-1)(x+1)(x-\sqrt{3})(x+\sqrt{3})$

VI. Profit = Revenue − Cost

Let x = selling price

Then demand $= D(x) = 4000 - \underbrace{500(x-5)}$ (buyers)

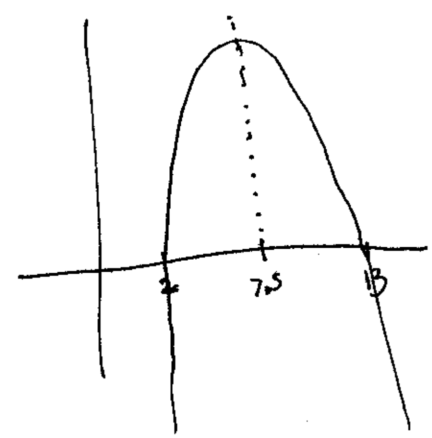
number of buyers lost as price increases

$= 4000-500x+2500$
$= 6500-500x$

Revenue = (selling price) × (demand)
$= x \cdot D(x) = x\cdot(6500-500x)$

Cost = (wholesale cost/item) × (number of items)
$= 2 \cdot D(x) = 2(6500-500x)$

$\therefore$ Profit $= x(6500-500x)-2(6500-500x)$
$=(x-2)(6500-500x)$
$=-500(x-2)(x-13)$



We don't really need to complete the square because we know the axis of symmetry will be half-way between the two x-intercepts: $\frac{2+13}{2}=7.5$

The new selling price should be \$7.50