Department of Mathematics Howard University
Math 015-007 (Precalculus)
Final Examination
Spring 2003

Instructions: Do all problems. Full credit will be given for complete solutions. Show all work!

1. [15 points] Find the amplitude, period, and phase shift of $y = -3\sin(2x + \pi/2)$. Use transformations to graph the function over $x \in [-\pi, \pi]$.

2. [20 points] Establish the identities:

(a) $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\left(\dfrac{\alpha+\beta}{2}\right)$

(b) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

3. [10 points] Use a half-angle formula to find the exact value of $\sin(-\pi/8)$.

4. [15 points] Psychologists sometimes use the formula $L(t) = A(1 - e^{-kt})$ to measure the amount L learned at time t. The number A represents the amount to be learned, and the number k measures the rate of learning. Suppose that the student has an amount A of 200 vocabulary words to learn. A psychologist determines that the student learned 20 vocabulary words after 5 minutes.

(a) Determine the rate of learning k.
(b) Approximately how many words will the student have learned after 10 minutes?
(c) How long does it take for the student to learn 180 words?

5 [15 points] Find an equation for the hyperbola with vertices at (0,-6) and (0,6) and asymptote the line $y = 2x$.

6. [15 points] Find the center, foci and vertices of the ellipse $2x^2 + 3y^2 - 8x + 6y + 5 = 0$. Then graph the equation.

7. [10 points] Find the vertex, focus, and directrix of the parabola $x^2 + 8x = 4y - 8$. Then graph the equation.

8. [30 points] Solve each equation for x.

(a) $8^{x^2-2x} = \frac{1}{2}$

(b) $\left(\frac{3}{5}\right)^{x} = 7^{1-x}$

(c) $\log_4 x + \log_4 (x-3) = 1$

9. [20 points] Compute the exact value (no calculators).

(a) $\sin\left(\cos^{-1}\left(\frac{5}{13}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$

(b) $\cos\left(2\tan^{-1}\left(\frac{4}{3}\right)\right)$

10. [20 points] A movie theatre charges \$9.00 for adults and \$7.00 for senior citizens. On a day when 325 people paid an admission, the total receipts were \$2495. How many who paid were adults? How many were senior citizens?

11. [20 points] Solve each triangle.

(a) $a = 3,\ \alpha = 10°,\ b = 4$

(b) $a = 1,\ b = 3,\ \gamma = 40°$

12. [10 points] Find the area of the triangle that has $a = 2,\ b = 3,\ \gamma = 40°$