HOWARD UNIVERSITY

DEPARTMENT OF MATHEMATICS: FINAL EXAMINATIONS.

MATH:007 : PRE-CALCULUS: WEDNESDAY, MAY $2^{\text{nd.}}$, 2007.

TIME: 4.00 P.M.—6.00 P.M.

ATTEMPT ANY EIGHT [[[8]]] PROBLEMS.

[[1]] **[25 Points]**

(*a*) Show that:

$$\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

and use it to find $\sin\frac{\pi}{16}$ and $\cos\frac{\pi}{16}$.

(*b*) The current I, in amperes, flowing through an AC (alternating current) circuit at time t is given by:

$$I = 220\sin(30\pi t + \frac{\pi}{6}),\ t \geq 0.$$

What is:

(*i*) the period? (*ii*) the amplitude ? (*iii*) the phase shift?

(*iv*) Graph this function over two periods.

[[2]]. **[25 Points]**

(*a*) Find the principal or present value P needed to get an amount $A = \$1000$ after one year at the interest rate of 12% compounded continuously.

(*b*) Find the exact value of the expression without the use of a calculator:

$$\cos\left[\tan^{-}\frac{4}{3} + \cos^{-1}\frac{12}{13}\right].$$

[[3]]. **[25 Points]**

(*a*) Find the exact value of the given expression without the use of a calculator:

$$4\csc\frac{3\pi}{4} - \cot(-\frac{\pi}{4}).$$

(*b*) An object P is travelling around a circle of radius $r = 2$ meters. If in 20 seconds the object travels 5 meters, what is its angular speed ω? What is its linear speed ν?

[[4]]. **[25 Points]**

(a) Use Fundamental Identities and/or the complementary Angle Theorem to find the exact value of the given expression without the use of a calculator:

$$\sec 35^\circ \csc 55^\circ - \tan 35^\circ \cot 55^\circ.$$

(b) Show that:

$$\cos(\tan^{-1} \nu) = \frac{1}{\sqrt{1+\nu}}.$$

[[5]] **[25 Points]**

(a) Do not use a calculator. Does

$$\sin^{-1}[\sin(\frac{2\pi}{2})] = \frac{2\pi}{2}?$$

For your answer, also say why or why not.

(b) The number of watts W provided by a space satellite's power supply after a period of d days is given by the function:

$$W(d) = 50e^{-0.004d}.$$

How much power will be available after 365 days.

[[6]] **[25 Points]**

(a) Let $P(-3, , -3)$ be a given point on the terminal side of an angle θ. Find the exact value of each of the six trigonometric functions of θ.

(b) Find the exact value of each of the remaining trigonometric functions of θ given that:

$$\cot\theta = \frac{4}{3}, \quad \cos\theta < 0.$$

[**Hint**: Use reference angle α.]

[[7]]. **[25 Points]**

(a) Show that:

$$\frac{1+\sin\alpha}{1-\sin\alpha} - \frac{1-\sin\alpha}{1+\sin\alpha} = 4\tan\alpha\sec\alpha.$$

(b) A skillet is removed from an oven with temperature $450°F$ and placed in a room of temperature $70°F$. After five minutes, the temperature of the skillet is $400°F$. How long will it be until its temperature is $150°F$?

[[8]] **[25 Points]**

(*a*) Given $\cot\theta = 2$, use trigonometric identities to find the exact value of :

$$(i) \quad \tan\alpha, \quad (ii) \quad \csc^2\alpha \quad (iii) \quad \tan(\frac{\pi}{2} - \alpha) \quad (iv) \quad \sec^2\alpha.$$

(*b*) Let the point $P(-2, 3)$ on the circle $x^2 + y^2 = r^2$ lie on the terminal side of a central angle θ. Find the values of the trigonometric functions:

$$\sin\theta, \; \cos\theta, \; \tan\theta, \; \csc\theta, \; \sec\theta, \; \text{and} \; \cot\theta.$$

[[9]] **[25 Points]**

(*a*) Solve the following system of equations using matrices (row operations). If the system has no solution, say that it is inconsistent;

$$\begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

(*b*) Find the exact value of the expression without using a calculator:

$$\sec[\sin^{-1}\frac{2\sqrt{5}}{5}].$$

[[10]] **[25 Points]**

(*a*) Write the equation of a sine function that has the given characteristics:
Amplitude: 3, Period: π and Phase shift: -2.

(*b*) Determine the amplitude, the Period and the Phase shift of the following function. Graph the function showing at least one period:

$$y = 3\cos(-2x + \frac{\pi}{2}).$$

(*c*) Given $\sec\alpha = 3$, $\csc\alpha < 0$.
Find

$$(i) \quad \sin\frac{\alpha}{2} \quad (ii) \quad \cos 2\alpha.$$

[[11]] **[25 Points]**

(*a*) The function:

$$f(x) = \frac{x^2 + 3}{3x^2}, \quad x > 0,$$

is one-to-one. Find its inverse and check your answer. State the domain of f and find its range.

(*b*) Solve:

$$\log_9 x + 3\log_3 x = 14.$$

(*c*) Write the given expression as a sum/or difference of logarithms. Express powers as factors:

$$\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right], \quad 0 < x < 1.$$

[[12]] [**25 Points**]

(*a*) Use the law of sines to solve the triangle ABC given that the angle at the vertex B, $\beta = 10°$, the side opposite this angle is $b = 2$ and the angle at vertex C, $\gamma = 100°$.

(*b*) Show that for any triangle ABC:

$$\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}},$$

where $s = \frac{1}{2}(a+b+c)$.

[**Hint:**Use Half-angle Formula and the Law of Cosines.]

[[13]] [**25 Points**]

(*a*) Solve the following system of equations using matrices (row operations). If the system has no solution, say that the system is inconsistent:

$$\begin{cases} 2x + y - 3z = 0 \\ -2x + 2y + z = -7 \\ 3x - 4y - 3z = 7 \end{cases}$$

(*b*) Solve the system of equations by substitution and elimination method. If the system has no solutions, say it is inconsistent.

$$\begin{cases} 3x + 3y = -1 \\ 4x + y = \frac{8}{3} \end{cases}$$

[[14]] [**25 Points**]

(*a*) Discuss the given equation of an ellipse; that is, find the center, foci,and vertices. Graph the equation.

$$x^2 + 9y^2 + 6x - 18y + 9 = 0.$$

(*b*) Find the center,transverse axis, vertices, foci, and asymptotes of the given hyperbola. Graph the equation.

$$2y^2 - x^2 + 2x + 8y + 3 = 0.$$