

Final Examination (2 hours)

Wednesday, December 10, 2003 (4pm to 6pm)

Please read instructions carefully:

Answer all questions. Each question is worth 14 points.

Show all work to justify your solutions for points to be awarded.

1. Let $f(x) = \frac{1}{x-1}$ for $x > 1$.

(a) Find the formula for $f^{-1}(x)$ and state the domain and range for the inverse function.(b) Find the exact value of $f(f^{-1}(2))$.

2. Solve the following equations:

(a) $2^{3x-2} = 3^{2x+1}$

(b) $\log x + \log(x-15) = 2$

(c) $(\sqrt[3]{2})^{2-x} = 2^{x^2}$

3. (a) Graph the function: $f(x) = \begin{cases} -e^x, & x < 0 \\ -e^{-x}, & x \geq 0 \end{cases}$. State the domain, range, and intercepts if any.(b) A radioactive substance is decaying exponentially. The amount of substance is reduced from 800 grams to 400 grams after 4 days. How much of the substance would remain after 10 days? (Hint: use the formula $A(t) = A_0 e^{kt}$, $k < 0$).4. Find the equation of the hyperbola which is centered at $(-2, -3)$ with one focus at $(-4, -3)$ and one vertex at $(-3, -3)$. State clearly the asymptotes, the second focus and vertex, and graph the hyperbola.

5. Solve the system of linear equations below and state whether the system is consistent or inconsistent, dependent or independent.

$$\begin{cases} x + 4y - 2z & = & 1 \\ 2x + 6y - z & = & 0 \\ -3x - 10y + 3z & = & -1 \end{cases}$$

6. A total of \$2000. is invested at 8% per annum compounded monthly.

(a) What would be the value of the investment after 4 years?

(b) How long would it take the investment to grow to \$3800?

7. Find the center, vertices, foci, and graph the following:

$$16x^2 + 4y^2 + 96x - 8y + 84 = 0.$$

8. Find the vertex, focus, and directrix of the parabola $y^2 + 4y - 4x + 16 = 0$. Sketch the graph of the parabola.

9. Find the inverse of the matrix: $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$.

State and show all your work and row operations in obtaining the inverse matrix.

10. Using the inverse found for question 9 or otherwise, solve the system:

$$\begin{cases} x - 2y + 3z = -1 \\ 2x - y + z = 2 \\ 4x + y + 2z = 1 \end{cases}$$

11. Graph the system of inequalities: $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 16 \\ 3x + 6y \leq 60 \end{cases}$. Find all vertices and label the graph.

12. Solve the nonlinear system of equations: $\begin{cases} x^2 + y^2 = 25 \\ y^2 - x = 5 \end{cases}$. Graph the system indicating the solutions of the system.

13. An arithmetic sequence is given by: 4, 7, 10, 13, ...

(a) Find the 55th term of the sequence.

(b) Find the sum of the first 47 terms.

14. Determine whether the infinite geometric series has a sum and if so find it.

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$$

15. (a) How many different 11-letter words (real or imaginary) can be formed from the letters in the word MATHEMATICS?

(b) A committee consisting of 2 males and 3 females is to be formed. If there are 5 males and 8 females to choose from, how many different committees are possible?