

HOWARD UNIVERSITY
Final Examination
College Algebra II (MATH-010), Fall 2005

Total points: 200

1. (a) Let $F(x) = \frac{3x}{x+2}$, find $F^{-1}(x)$ and state the domain and range of $F(x)$. What is the value of $F^{-1}(F(4))$.
(b) Let $f(x) = x^2$ for $x \geq 0$, and $g(x) = -\sqrt{x}$, is $g(x) = f^{-1}(x)$? Justify your answer.
(c) Let $h(x) = \frac{3x+4}{2x-4}$ show that $h^{-1}(x) = h(x)$.

2. Graph the following functions indicating all intercepts and asymptotes.

(a) $f(x) = 1 + \ln(x+1)$.

(b) $g(x) = e^{-|x|}$, that is,

$$g(x) = \begin{cases} e^x & \text{if } x \geq 0 \\ e^{-x} & \text{if } x < 0. \end{cases}$$

3. An island in the pacific experienced an exponential growth in population. Given that the population doubled over the past 20 years, what would the population be 5 years from now, if the total number of islanders has reached 100,000 people?
4. Suppose you invest \$1000 in a venture that yields 12% compounded quarterly. How much time will it take for your investment to grow to \$4000.
5. Solve for x and simplify when possible.

(i) $9^{\frac{1}{2}x^2-1} + 1 = 4$

(ii) $5^{2x-1} = 2^{1-x}$

(iii) $\log_2(x+4) + \log_2(x-2) = 4$.

6. Determine the type of conic section (hyperbola, parabola, or ellipse) given by

$$9x^2 + 4y^2 + 54x - 8y + 49 = 0$$

and write down the coordinates of the center, vertices and foci.

7. Graph the parabola $y^2 - 2y - 8x + 9 = 0$, indicating the vertex, intercept(s) and latus rectum. Also draw the directrix and the axis of symmetry.

8. Let

$$A = \begin{pmatrix} 1 & 6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{pmatrix}$$

(a) Compute the product $A \cdot A$

(b) Use the result in part A, or any other method to solve the system of equations

$$x - 6y + 3z = 5$$

$$2x - 7y + 3z = 7$$

$$4x - 12y + 5z = 11$$

(c) Compute the determinant of A (denoted by $\det(A)$). Can you explain why you would expect such a result, in view of part (a) and the fact that $\det(A^{-1}) = 1/\det(A)$.

9. Solve the system of equations:

$$x + y + 2z = 1$$

$$x - y - z = -1$$

$$2x + z = 0$$

10. (a) What is the value of the determinant of a matrix in echelon form, where all the rows are non-zero.
 (b) More generally, what is the value of the determinant of a matrix with zeros below the main diagonal. (Justify your answers).
11. (a) Graph the region that satisfies the constraints

$$x \geq 0, \quad y \geq 0 \quad x \leq 3 \quad y \leq 4 \quad 5x + 4y \geq 20$$

- (b) Find the maximum and minimum values of the objective function

$$z = 10x - 8y + 28$$

subject to the constraints in part (a) and the coordinates x and y at which they occur.

- (c) Graph the region satisfying the two inequalities:

$$\begin{cases} x^2 + y^2 \leq 4 \\ x^2 + 2y \geq 4 \end{cases}$$

12. (a) Find the sum of the first 50 terms of the series given by the general formula $\{5n - 1\}$.
 (b) Find the sum of all the odd numbers from 7 to 493 inclusive.
 (c) Suppose that $\{a_n\}$ is an arithmetic sequence such that $a_7 = 31$ and $a_{20} = 96$, find a general formula for a_n .
13. (a) Find the sum of the series with general term $a_n = 3^{n-1}/4^n$.
 (b) Find and simplify the third term in the binomial expansion of $(x - 3y)^5$.
14. (a) Find t so that $t + 3$, $2t + 1$, and $5t + 2$ are consecutive terms of an arithmetic sequence.
 (b) Evaluate $S_{21} = \sum_{k=1}^{21} k$.
 (c) Write down the n^{th} term of the sequence $\{a_n\}$ suggested by the pattern:

$$0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$$

in other words, find a general formula for a_n .