Department of Mathematics **Howard University**

College Algebra II (Math-010)- Final Examination

April 28, 2004

Do all questions. Show all work.

The time for this examination is two (2) hours.

1. [10 Points]

The function

$$f(x) = \frac{2x+1}{x-1}, \quad x \neq 1$$

is one-to-one.

a. Find a formula for $f^{-1}(x)$.

b. State the domain of f and find its range using f^{-1}.

2. [20 Points]

a. Sketch the graphs of $f(x) = 2^x$ and $g(x) = \log_2 x$, using the same coordinate axes.

b. What is the relationship of g to f?

c. Give the x-intercept, the domain, and the range of g.

3. [15 Points]

Solve each equation:

a. $\log_x(\frac{1}{16}) = \frac{4}{3}$

b. $\log_2 x + \log_2(x+2) = \log_2(6x+5)$

4. [10 points]

How much should be invested now to have \$800 after $3\frac{1}{2}$ years, if the account pays 7% compounded monthly?

5. [20 Points]

Given that $\log_b u = 1.938$ and $\log_b v = 0.132$ $(b > 0, b \neq 1)$, find each of the following:

a. $\log_b(\frac{u}{v})$

b. $\log_b \sqrt{uv}$

c. $\log_b(uv)$

d. $\frac{\log_b u}{\log_b v}$

e. $(\log_b u)(\log_b v)$

6. [15 Points]

a. If $A = \begin{bmatrix} -4 & -2 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -4 \\ -3 & 8 \end{bmatrix}$, find $3A - 2B$.

b. If $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 2 \end{bmatrix}$, find each of the following if possible:

(i) AB

(ii) BA

7. [15 Points]
Use any valid method you wish to solve the system of equations:

$$\begin{aligned} 2x + 3y + z &= 14 \\ 3x - 4y - 2z &= -30 \\ 5x + 7y + 3z &= 32 \end{aligned}$$

8. [15 Points]
The matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

is nonsingular. Find its inverse A^{-1}.

9. [15 Points]
Given that

$$\begin{vmatrix} 5 & 3 & 1 \\ 1-x & 2 & 4 \\ 4 & 3 & 2 \end{vmatrix} = 6,$$

solve for x.

10. [20 Points]

Maximize and minimize the objective function $P(x, y) = 9x + 13y$ subject to the following constraints:

11. [10 Points]
Solve the system of equations and sketch the graph of each equation on the same coordinate plane.

$$\begin{aligned} y &= |3x| \\ y &= x^2 + 2 \end{aligned}$$

12. [10 Points]
Find the vertex, focus, and directrix of the parabola given by the equation:

$$x^2 + 6x - 4y + 17 = 0\,.$$

13. [10 Points]
A conic section has the equation

$$\frac{(x-3)^2}{4} + \frac{(y+1)^2}{25} = 1$$

a. What type of conic section is this? (parabola, ellipse, or hyperbola)

b. What is its center?

c. What are its foci?

d. What are its vertices?

e. Graph the conic section using the information you have given.

14. [15 Points]
Determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference and the 51st term; if it is geometric, find the common ratio and the 8th term.

a. $2, -2, -6, -10, \cdots$

b. $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \cdots$