Final Examination

1. [20pts] True or False

____  If \( f \) is a function, then the inverse \( f^{-1} \) of \( f \) is \( f^{-1}(x) = \frac{1}{f(x)} \)

____  \( 10^{\log_{10} x} = x \)

____  \( \log_\alpha (\alpha^x) = x \)

____  \( \log_\alpha (\alpha^4) = 4 \)

____  \( \log (5 \cdot 4) = \log 5 + \log 4 \)

____  \( \log \left( \frac{5}{3} \right) = \frac{\log 5}{\log 3} \)

____  \( \log (5 - 3) = \log 5 - \log 3 \)

____  If \( A \) and \( B \) are matrices for which \( A \cdot B \) and \( B \cdot A \) are defined, then \( A \cdot B = B \cdot A \)

____  Every square matrix with determinant non zero has an inverse matrix.

____  If \( A^{-1} \) is the inverse of the matrix \( A \), then \( A^{-1} = \frac{1}{A} \)

2. [12pts] Given the function \( f(x) = \frac{5x - 3}{2x + 1} \), determine whether it is one-to-one. If it is one-to-one, find a formula for the inverse.

3. [12pts] Solve each equation:
   (a) \( \frac{1}{2^x+1} = 64 \)
   (b) \( \log_5(x + 4) + \log_5(x - 4) = 2 \)

4. [20pts] (a) Find the inverse \( A^{-1} \) of the matrix \( A = \begin{bmatrix} 3 & 1 \\ 2 & -2 \end{bmatrix} \)

(b) Use \( A^{-1} \) to solve the system
\[
\begin{align*}
3x + y &= 9 \\
2x - 2y &= 14
\end{align*}
\]

(c) Solve the above system using Cramer's rule.
5. [12pts] Let \( f(x) = \left( \frac{1}{3} \right)^x \). Sketch the graph of \( f \) and find the inverse \( f^{-1} \) of \( f \).

6. [12pts] Evaluate the determinant \[
\begin{vmatrix}
2 & -2 & 2 \\
3 & 1 & 0 \\
2 & -1 & 1
\end{vmatrix}
\]

7. [12pts] If \$6000\) is invested at the annual interest rate of 7\%, compounded quarterly, what is the investment worth in 10 years?

8. [12pts] Find \( A \cdot B \) for \( A = \begin{bmatrix} -4 & -1 & 2 \\ 3 & 8 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} -2 & 1 \\ 7 & 0 \\ 4 & -6 \end{bmatrix} \). Can you find \( B \cdot A \)?

9. [20pts] Find the maximum and minimum values of the function \( F = 16x + 14y \) subject to the constraints \( x \geq 0, \ y \geq 0 \)
\begin{align*}
&3x + 2y \leq 12 \\
&7x + 5y \leq 29
\end{align*}

10. [12pts] Decompose \( \frac{x+7}{(x-3)(x+2)} \) into partial fractions.

11. [12pts] Let \( \{ a_n \} \) be an arithmetic sequence such that \( a_{21} = 7 \) and \( a_{13} = 3 \), find the first term \( a_1 \) and the common difference \( d \).

12. [12pts] Find the sum \( \sum_{k=1}^{\infty} 8\left( \frac{3}{4} \right)^{k+1} \)

13. [12pts] Find the 7th term in the expansion of \( (3a + b)^{11} \)

14. [20pts] Find the center, the vertices, the foci of the ellipse \( 16x^2 + 4y^2 + 96x - 8y + 84 = 0 \), and graph the ellipse.

15. [20pts] Find the vertex, the focus, the directrix and draw the graph of \( (y-3)^2 = -20(x+2) \).