

# Howard University Department of Mathematics

## Math 026 (Applied Calculus) Final Examination

December 11, 2002

Time: Two hours

Show all of your work.

1. [10 Points] Find the indicated limits.

(a)  $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x}$

(b)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 2}{4x^2 + 3x - 1}$

2. [15 Points] Use the techniques of differentiation to find the first derivative of each of the following.

(a)  $y = f(x) = 4x^2(2x^2 + 3)$

(b)  $y = f(x) = e^{2x} + 6$

(c)  $y = \frac{x^2 - 2x}{3x + 1}$

3. [15 Points] Assuming that the equation  $y^2 + x^2y = 3x^2$  defines  $y$  as a differentiable function of  $x$  near the point  $(2, 2)$ , find:

(a)  $\frac{dy}{dx}$  by using implicit differentiation;

(b) the slope of the tangent line to the graph at the point  $(2, 2)$ .

4. [20 Points] Determine where the graph of the function  $f(x) = x^3 - 3x^2 - 24x + 1$  is increasing, where it is decreasing, where it is concave upward, and where it is concave downward. Find the relative extrema and the inflection point and sketch the graph.

5. [15 Points] An electronics company estimates that the cost (in dollars) of producing  $x$  components used in electronic toys is given by  $C(x) = 200 + 0.5x + 0.002x^2$ .

(a) Find the cost of producing 50 components.

(b) Find the marginal cost of producing 50 components.

(c) Find the actual cost of producing the 50<sup>th</sup> component.

6. [10 Points] The gross annual earnings of a company were  $f(t) = (10t^2 + t + 236)^{\frac{1}{2}}$  thousand dollars  $t$  years after its formation in January, 1998. At what rate were the gross annual earnings of the company growing in January 2002?

7. [15 Points] A producer of fashion watches determines that the demand for one of its models is related to its selling price by the equation  $p(x) = 50 - \frac{1}{40}x$  where  $x$  is the number of sales per week and  $p(x)$  is the selling price (in dollars).
- Find the manufacturer's weekly revenue  $R(x)$  as a function of  $x$ .
  - Find the marginal revenue.
  - Evaluate the marginal revenue at a demand level of  $x = 100$ .
8. [15 Points] Find the solution of the differential equation  $\frac{dy}{dx} = x^2y$  for which  $y = 1$  when  $x = 3$ .
9. [10 Points] Evaluate the following antiderivatives.
- $\int (x^2 + \sqrt{x}) dx$
  - $\int x^4 \ln(2x) dx$
10. [5 Points] Evaluate the definite integral  $\int_0^2 3x^2 \sqrt{x^3 + 1} dx$ .
11. [10 Points] At a certain factory, the marginal cost is  $5q^2 - 2q$  dollars per unit when the level of production is  $q$  units. By how much will the total manufacturing cost increase if the level of production is increased from 3 to 6 units?
12. [10 Points] Let  $z = f(x, y) = x^2ye^{2y}$ . Compute each of the following.
- $\frac{\partial z}{\partial x} = f_x(x, y)$
  - $\frac{\partial^2 z}{\partial y \partial x} = f_{xy}(x, y)$
13. [15 Points] Find the critical point of  $f(x, y) = x^2 + xy + y^2 - 3x$  and classify it as a relative maximum, relative minimum, or saddle point.
14. [20 Points] Find the maximum and minimum values of  $f(x, y) = xy$  subject to the constraint that  $x^2 + 4y^2 = 2$ .
15. [15 Points] Evaluate  $\iint_R xy dA$  where  $R = \{(x, y) : 0 \leq x \leq 1 \text{ and } -1 \leq y \leq 0\}$ .