Department of Mathematics Howard University

Applied Calculus (Math-026)- Final Examination

December 10, 2003

Do all questions. Show all work.
The time for this examination is two (2) hours.

1. [20 Points]
Use techniques of differentiation to find the derivative of each of the following:

a. $y = x^2\, e^{3x}$

b. $y = \frac{x^2-4}{x-2}$

c. $y = (x^2 - 6x + 2)^5$

d. $y = \ln(2x + 1)$

2. [15 Points]
If $4x^2 - 2y^2 = 9$, use implicit differentiation to find (a) $\frac{dy}{dx}$ and (b) $\frac{d^2y}{dx^2}$.
Simplify each result.

3. [10 Points]
Find the indicated limits:

a. $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$

b. $\lim_{x\to\infty} \frac{x-3}{2x+x^3}$

4. [10 Points]
The gross annual earnings of a certain company were $f(t) = \sqrt{10t^2 + 2t + 52}$ thousand dollars t years after its formation in January 1988. At what rate were the gross annual earnings of the company growing in January 1992?

5. [20 Points]
Let $f(x) = 2x^3 + 3x^2 - 12x + 3$.
Determine where the function is increasing, decreasing, concave upward, and concave downward. Find the relative extrema and inflection point and sketch the graph.

6. [20 Points]
The total cost, $C(x)$, of producing x units of a particular commodity is given by $C(x) = \frac{1}{4}(x^2 + 3x + 52)$ dollars and $p(x) = \frac{1}{4}(18 - x)$ dollars is the price at which all x units will be sold.
a. Find the marginal cost and the marginal revenue.
b. Use marginal cost to estimate the cost of producing the 3rd unit.
c. Find the actual cost of producing the 3rd unit.
d. Use the marginal revenue to estimate the revenue derived from the sale of the 3rd unit.

7. [20 Points]
An efficient study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8 a.m. will have produced $Q(t) = -t^3 + 6t^2 + 24t$ units t hours later.
a. Compute the worker's rate of production at 11:00 a.m.
b. At what rate is the worker's rate of production changing with respect to time at 11:00 a.m.?

8. [10 Points]
When electric blenders are sold for p dollars each, manufacturers will supply $S(p) = p - 10$ blenders to local retailers while the local demand will be $D(p) = \frac{5600}{p}$ blenders.
At what market price will the manufacturer's supply of electric blenders be equal to the consumer's demand for blenders? How many blenders will be sold at this price?

9. [10 points]
Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = \sqrt{\frac{2x^2+1}{x^2+2}}$. Simplify your answer.

10. [10 Points]
Use integration by substitution to find:

a. $\int 2x\, e^{x^2+2}\, dx$

b. $\int \frac{10x^3-5x}{\sqrt{x^4-x^2+6}}\, dx$

11. [10 Points]
Use integration by parts to compute:

(a) $\int x\, e^{2x}\, dx$

OR

(b) $\int x \ln 2x\, dx$.

12. [10 Points]
Evaluate each of the following:

a. $\int_0^3 x^2\, dx$

b. $\int_1^3 \frac{1}{x}\, dx$

13. [10 Points]
Find the area of the region bounded by the curves $y = x^2$ and $y = 2x$. Give a sketch of the region.

14. [15 Points]
For $f(x, y) = 6x^2 - 2x^2y^2 + y^2 + 5$, find the following:

(a) f_x (b) f_y (c) f_{xx}
(d) f_{yy} (e) f_{xy} (f) f_{yx}

15. [10 Points]
Find the general solution of the differential equation $\frac{dy}{dx} = \frac{3x^2+2}{y}$.