

Math 026 Applied Calculus – Final Exam

Each problem is worth 20 points. Answer any 10 problems. Only 10 will be counted for your grade. Show all work!

1. Find the following limits, or indicate that the limit does not exist.

(a) $\lim_{x \rightarrow 16} \frac{16-x}{4-\sqrt{x}}$.

(b) $\lim_{x \rightarrow 2} \frac{x^2+5x-14}{x^2-4}$.

(c) $\lim_{x \rightarrow \infty} \frac{3x^2-2x+8}{6x^2+5x+1}$.

2. Find the derivative $f'(x)$ in each case.

(a) $f(x) = x^5 + 2x^{3/7} - \frac{5}{\sqrt{x}} + 10$.

(b) $f(x) = xe^{2x} + \ln x + 3$.

(c) $f(x) = (9x^2 - 2)^6$.

3. Let $f(x) = 2x^3 + 3x^2 - 12x + 1$.

(a) Find $f'(x)$ and determine where f is increasing and where f is decreasing.

(b) Use the information in (a) to sketch the graph of f .

4. Find the absolute maximum and minimum of $f(x) = x^4 - 2x^2 - 1$ in the interval $0 \leq x \leq 2$.

5. The total cost, $C(x)$, of producing x units of a particular commodity is given by $C(x) = \frac{1}{5}x^2 + 10x + 30$ and $p(x) = \frac{1}{2}(50 - x)$ is the price at which all x units will be sold.

(a) Find the marginal cost and the marginal revenue.

(b) Use marginal cost to estimate the cost of producing the 5th unit.

(c) Find the actual cost of producing the 5th unit.

(d) Use marginal revenue to estimate the revenue derived from the sale of the 5th unit.

6. Given $f(x) = 3x^2 + \ln x$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Write an equation for the tangent line to the graph of $f(x)$ at the point $(1, 3)$.

7. The position of a particle $s(t)$ moving on a straight line at time t seconds is given by $s(t) = t^2 - 6t + 1$.
- Find the velocity and acceleration of the particle at time t .
 - Find all times in the first 8 seconds when it is stationary.
 - Find the total distance traveled by the particle between times $t = 0$ and $t = 8$.
8. Use implicit differentiation to find $\frac{dy}{dx}$.
- $x^2y = 1$.
 - $(x + xy^4)^3 = x + \ln y$.
9. Suppose \$5,000 is at an annual interest rate of 8%. Compute the balance after 6 years if the interest is compounded
- quarterly
 - continuously
10. Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = \frac{(x+e^{2x})^2 e^{5x}}{(1+x)^{2/3}}$. Simplify your answer.
11. Find the following integrals.
- $\int (x^2 + 3x + 5)dx$.
 - $\int x^2(x^3 - 6)^4 dx$.
 - $\int (3e^{4x} - e^x + \frac{1}{x})dx$.
12. Evaluate the following integrals.
- $\int_1^2 (3x^2 - 4x + \frac{1}{x^2})dx$.
 - $\int_0^3 xe^{x^2+1} dx$.
 - $\int_2^4 \frac{1}{x} dx$.
13. Integrate by parts: $\int xe^{-x} dx$.
14. Solve the differential equation $\frac{dy}{dx} = \frac{x}{y^2}$ with the initial condition $y = 3$ when $x = 4$.
15. Compute all second order partial derivatives (including mixed partials) of $f(x) = xye^x$.