Final Exam
015-156
Fall 2001

Please do all problems. Points are written to the left of each problem,

24 pts 1. Evaluate the following limits. If the limit does not exist, say so.

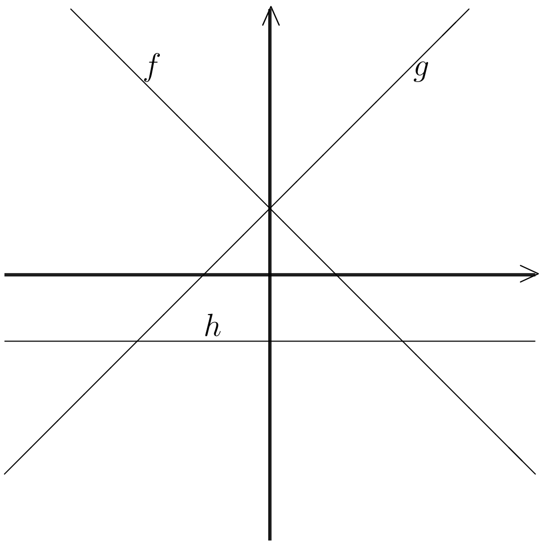
(a) $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

(b) $\lim_{x \to 0^-} \frac{x^2 + x}{|x|}$

(c) $\lim_{x \to c} \frac{\sin^3(x) - \sin^3(c)}{x - c}$

12 pts 2. Recall that there are two equivalent definitions of the derivative, namely $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ and $f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$. Let $f(x) = \frac{1}{2x}$. Show that $f'(x) = -\frac{1}{2x^2}$ by using one of the two definitions of the derivative.

12 pts 3. Shown below are the graphs of three functions, f, g, and h. One of these is the derivative of one of the others. Indicate which function is the derivative of which other function and justify your answer.



12 pts 4. Find the absolute maximum and minimum of the function $f(x) = x - \sin x$ on the interval $[0, \pi]$.

12 pts 5. Acceleration due to gravity near the surface of Mars is 3.61 m/sec^2. If a rock is thrown upward from the surface with an initial velocity of 5 m/sec, what will be its maximum altitude?

32 pts 6. Differentiate the following.

(a) $f(x) = e^{2x} + \sin x$

(b) $g(x) = \dfrac{x^3 + x}{x^2 + 1}$

(c) $y = \sin^{-1}(x + 2)$

(d) $h(x) = \displaystyle\int_1^x \frac{\sin t}{1 + \cos t}\, dt$

24 pts 7. Let $f(x) = x^{1/3}(x - 1)$. Find the intervals on which f is

(a) increasing (b) decreasing (c) concave up (d) concave down

12 pts 8. (a) Find the area between the graph of $y = \sqrt{x}$ and the x-axis on the interval $[0, 1]$.

(b) Find the area between the graph of $y = x^2$ and the x-axis on the interval $[0, 1]$.

(c) Add the results from (a) and (b) and give a *geometric* explanation of this answer. [Hint: $\displaystyle\int_0^1 x^2\, dx = \int_0^1 y^2\, dy$.]

12 pts 9. Find an equation of the line tangent to the graph of $x^2y^3 + x^3y = 6$ at the point $(-1, 2)$.

12 pts 10. A fire is burning on a hay field. The burned area is in the shape of a circle. If the burned area is increasing at the rate of 10 ft^2/sec, how fast is the radius increasing when the radius is 100 ft?

18 pts 11. Find the indicated antiderivative.

(a) $\displaystyle\int \frac{x^2 + 1}{x}\, dx$

(b) $\displaystyle\int 2xe^{x^2}\, dx$

18 pts 12. Evaluate the definite integral.

(a) $\displaystyle\int_1^e \frac{1}{x} \ln x\, dx$

(b) $\displaystyle\int_{-1/4}^{\sqrt{3}/4} \frac{1}{\sqrt{1 - 4x^2}}\, dx$