Final Exam  
Math 156 Calculus I  
Spring 2003

1. [27 points]. Evaluate the following limits if they exist. If they do not exist, say so and explain your reasons for concluding that they do not exist.

(a) \( \lim_{x \to 0} \frac{\sin(11x)}{x} \)

(b) \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \)

(c) \( \lim_{x \to 2} \frac{|x - 2|}{x - 2} \)

2. [9 points]. Find values of \( a \) and \( b \) so that the following function is continuous everywhere.

\[
f(x) = \begin{cases} 
  x + 4 & \text{if } x < -2 \\
  ax + b & \text{if } -2 \leq x \leq 2 \\
  x - 4 & \text{if } 2 < x.
\end{cases}
\]

3. [9 points]. Use the definition of the derivative to show that \( f'(x) = 3x^2 - 3 \) if \( f(x) = x^3 - 3x \).

4. [12 points]. Show that \( f(x) = \begin{cases} 
  x^2 - 5 & \text{if } x \leq 1 \\
  x - 5 & \text{if } 1 < x
\end{cases} \) is continuous but not differentiable at 1.

5. [36 points]. Differentiate the following.

(a) \( f(x) = x^2 \sin x + \cos x \)

(b) \( y = \frac{e^x + 2}{e^x - 1} \)

(c) \( f(x) = \frac{1}{(x - 3)^5} \)

(d) \( f(x) = \int_0^x t \sin \left( \frac{1}{t^3 - 2} \right) \, dt \)

6. [9 points]. Find an equation of the line tangent to the graph of \( x^4 + 2xy + 4x = 7 \) at the point \( (1, 1) \).
7. [12 points]. A rectangular field is to be enclosed and divided into three rectangular subplots as shown in the diagram below. A total of 600 meters of fence is to be used. Find the dimensions that maximize the total enclosed area.

8. [11 points]. Find all relative extrema of \( f(x) = x(x - 1)^2 \).

9. [11 points]. Find the maximum and minimum values of \( f(x) = 2x^3 + x^2 - 4x + 1 \) on the interval \([0, 1]\).

10. [9 points]. Let \( f(x) = \frac{1}{4}x^2 + 1 \). Show that \( f \) satisfies the hypotheses of the Mean Value Theorem on the interval \([-1, 4]\) and find a point \( c \) in the open interval \((-1, 4)\) which satisfies the conclusion of the Mean Value Theorem.

11. [10 points]. A particle moves with a velocity of \( v(t) = t^2 - 8t \) meters per second along an \( s \)-axis. Find the distance traveled by the particle during the period \( 0 \leq t \leq 10 \).

12. [9 points]. Evaluate the following indefinite integral. \( \int 2 \sin x \sqrt{\cos x + 1} \, dx \)

13. [27 points]. Evaluate the following definite integrals.
   \[
   \begin{align*}
   (a) & \quad \int_{-5}^{5} (2x^3 + 3x^2 + 7x) \, dx \\
   (b) & \quad \int_{1}^{2} \frac{e^{3/x}}{x^2} \, dx \\
   (c) & \quad \int_{-2/3}^{2\sqrt{3}/3} \frac{1}{4 + 9x^2} \, dx
   \end{align*}
   \]

14. [9 points]. Solve the equation \( e^x(e^x - 1) = 6 \).