

Final Exam
Math 156 Calculus I
Spring 2003

1. [27 points]. Evaluate the following limits if they exist. If they do not exist, say so and explain your reasons for concluding that they do not exist.

(a) $\lim_{x \rightarrow 0} \frac{\sin(11x)}{x}$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(c) $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$

2. [9 points]. Find values of a and b so that the following function is continuous everywhere.

$$f(x) = \begin{cases} x + 4 & \text{if } x < -2 \\ ax + b & \text{if } -2 \leq x \leq 2 \\ x - 4 & \text{if } 2 < x. \end{cases}$$

3. [9 points]. Use the definition of the derivative to show that $f'(x) = 3x^2 - 3$ if $f(x) = x^3 - 3x$.

4. [12 points]. Show that $f(x) = \begin{cases} x^2 - 5 & \text{if } x \leq 1 \\ x - 5 & \text{if } 1 < x \end{cases}$ is continuous but not differentiable at 1.

5. [36 points]. Differentiate the following.

(a) $f(x) = x^2 \sin x + \cos x$

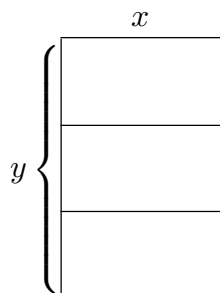
(b) $y = \frac{e^x + 2}{e^x - 1}$

(c) $f(x) = \frac{1}{(x - 3)^5}$

(d) $f(x) = \int_0^x t \sin\left(\frac{1}{t^3 - 2}\right) dt$

6. [9 points]. Find an equation of the line tangent to the graph of $x^4 + 2xy + 4x = 7$ at the point $(1, 1)$.

7. [12 points]. A rectangular field is to be enclosed and divided into three rectangular subplots as shown in the diagram below. A total of 600 meters of fence is to be used. Find the dimensions that maximize the total enclosed area.



8. [11 points]. Find all relative extrema of $f(x) = x(x - 1)^2$.
9. [11 points]. Find the maximum and minimum values of $f(x) = 2x^3 + x^2 - 4x + 1$ on the interval $[0, 1]$.
10. [9 points]. Let $f(x) = \frac{1}{4}x^2 + 1$. Show that f satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 4]$ and find a point c in the open interval $(-1, 4)$ which satisfies the conclusion of the Mean Value Theorem.
11. [10 points]. A particle moves with a velocity of $v(t) = t^2 - 8t$ meters per second along an s -axis. Find the distance traveled by the particle during the period $0 \leq t \leq 10$.
12. [9 points]. Evaluate the following indefinite integral. $\int 2 \sin x \sqrt{\cos x + 1} dx$
13. [27 points]. Evaluate the following definite integrals.
- (a) $\int_{-5}^5 (2x^3 + 3x^2 + 7x) dx$
- (b) $\int_1^2 \frac{e^{3/x}}{x^2} dx$
- (c) $\int_{-2/3}^{2\sqrt{3}/3} \frac{1}{4 + 9x^2} dx$
14. [9 points]. Solve the equation $e^x(e^x - 1) = 6$.