Please do all problems. Points are written to the left of each problem.

10 pts 1. The base of a solid is the region bounded by the ellipse \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \). Find the volume of the solid if the cross sections perpendicular to the \( x \)-axis are equilateral triangles. (Hint: The area of an equilateral triangle of side \( s \) is \( \frac{\sqrt{3}}{4} s^2 \).)

10 pts 2. Find the volume generated when the region bounded by the graphs of \( y = 2x - x^2 \) and \( y = 0 \) is revolved around the \( y \)-axis.

10 pts 3. A force of 2 Newtons was required to stretch a spring from its natural length of 30 centimeters to a length of 40 centimeters. How much work was done in stretching the spring to that length?

30 pts 4. Evaluate the following integrals:
   (a) \( \int \frac{x - 3}{x^2(x + 1)} \, dx \)
   (b) \( \int \frac{x^2}{1 + x^2} \, dx \)
   (c) \( \int \sin^3 x \sec x \, dx \)

10 pts 5. Determine whether the improper integral \( \int_1^2 \frac{1}{\sqrt{x - 1}} \, dx \) converges or diverges, giving a reason for your answer. If it converges, find its value.

30 pts 6. Determine whether the following series converge or diverge, giving reasons for your answers.
   (a) \( \sum_{k=1}^{\infty} \frac{(k + 1)(k + 2)}{k(k - 2)(k + 3)} \)
   (b) \( \sum_{k=1}^{\infty} \left( \frac{2k + 1}{3k - 4} \right)^k \)
   (c) \( \sum_{k=2}^{\infty} \frac{1}{k \ln k} \)
7. Determine whether the following series converge absolutely, converge conditionally, or diverge, giving reasons for your answers.

(a) \[ \sum_{k=0}^{\infty} \frac{(-1)^k k}{k^2 + 2} \]

(b) \[ \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(k+1)(k+2)} \]

8. Determine the interval of convergence of each of the following power series.

(a) \[ \sum_{k=0}^{\infty} \frac{k}{2k+1} (x-2)^{2k+1} \]

(b) \[ \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} (x+3)^k \]

(c) \[ \sum_{k=0}^{\infty} \frac{2^k}{(2k)!} x^k \]

9. Find the MacLaurin series expansion of \( \frac{x}{(1-2x)^2} \).

10. Find the Taylor series expansion of \( \sin(\pi x) \) around the point \( x = 1 \).

11. Evaluate \( \int_0^1 \sin(x^3) \, dx \) to three decimal point accuracy. (You will probably find it helpful to recall that for all \( x \), \( \sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \)).

12. Let \( C \) be the curve parametrized by the equations

\[ x(t) = t^2 - 2t \]
\[ y(t) = t^3 - 3t^2 + 2t . \]

Find the points at which the curve has a vertical tangent.

13. Find the area inside the graph of \( r = 4 \) and to the right of the graph of \( r = 2 \sec \theta \).