Final Exam

Math 157 – Calculus II

Spring, 2002

Please do all problems. Points are written to the left of each problem.

10 pts 1. The base of a solid is the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Find the volume of the solid if the cross sections perpendicular to the x-axis are equilateral triangles. (Hint: The area of an equilateral triangle of side s is $\frac{\sqrt{3}}{4}s^2$.)

10 pts 2. Find the volume generated when the region bounded by the graphs of $y = 2x - x^2$ and $y = 0$ is revolved around the y-axis.

10 pts 3. A force of 2 Newtons was required to stretch a spring from its natural length of 30 centimeters to a length of 40 centimeters. How much work was done in stretching the spring to that length?

30 pts 4. Evaluate the following integrals:

(a) $\displaystyle\int \frac{x-3}{x^2(x+1)}\,dx$

(b) $\displaystyle\int \frac{x^2}{1+x^2}\,dx$

(c) $\displaystyle\int \sin^3 x \sec x\,dx$

10 pts 5. Determine whether the improper integral $\displaystyle\int_1^2 \frac{1}{\sqrt[3]{x-1}}\,dx$ converges or diverges, giving a reason for your answer. If it converges, find its value.

30 pts 6. Determine whether the following series converge or diverge, giving reasons for your answers.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)(k+2)}{k(k-2)(k+3)}$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{2k+1}{3k-4}\right)^k$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

20 pts 7. Determine whether the following series converge absolutely, converge conditionally, or diverge, giving reasons for your answers.

(a) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{k}{k^2+2}$

(b) $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(k+1)(k+2)}$

30 pts 8. Determine the interval of convergence of each of the following power series.

(a) $\displaystyle\sum_{k=0}^{\infty}\frac{k}{2k+1}(x-2)^{2k+1}$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{k}}(x+3)^k$

(c) $\displaystyle\sum_{k=0}^{\infty}\frac{2^k}{(2k)!}x^k$

10 pts 9. Find the MacLaurin series expansion of $\dfrac{x}{(1-2x)^2}$.

10 pts 10. Find the Taylor series expansion of $\sin(\pi x)$ around the point $x=1$.

10 pts 11. Evaluate $\displaystyle\int_0^1 \sin(x^3)\,dx$ to three decimal point accuracy. (You will probably find it helpful to recall that for all x, $\sin x = \displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}$.)

10 pts 12. Let C be the curve parametrized by the equations

$$x(t) = t^2 - 2t$$
$$y(t) = t^3 - 3t^2 + 2t\,.$$

Find the points at which the curve has a vertical tangent.

10 pts 13. Find the area inside the graph of $r=4$ and to the right of the graph of $r = 2\sec\theta$.