1. [20pt.]
(a) Find the radius of convergence and the interval of convergence of the series:
\[ \sum_{k=0}^{\infty} \frac{(-3)^k x^k}{\sqrt{k + 1}}. \]

(b) Use any tests to determine whether the following series converge or diverge:
\[ (i) \sum_{k=1}^{\infty} \frac{\ln k}{k\sqrt{k}} \quad (ii) \sum_{k=1}^{\infty} \frac{k^2}{e^k}. \]

2. [20pt.]
(a) Use a suitable substitution and a reduction formula to evaluate: \( \int xe^{-\sqrt{x}} \, dx. \)

(b) Find the area of the surface generated by revolving the parametric curve:
\[ x = \cos^2 t, \quad y = \sin^2 t, \quad 0 \leq t \leq \frac{\pi}{2}, \]
about the \( y \)-axis.

3. [20pt.] Evaluate the following Integrals:
\[ (a) \int_{0}^{\infty} \frac{dx}{a^2 + bx^2}, \quad a > 0, \quad b > 0. \quad (b) \int \frac{\cos x}{\sin^2 x + 4 \sin x - 5} \, dx. \]

4. [20pt.]
(a) Find the total arclength of the cardioid: \( r = 1 + \cos \theta \), as \( \theta \) varies from \( \theta = 0 \) to \( \theta = 2\pi \).
(b) Find the area of the region bounded by one loop of the graph of the polar equation: \( r = 2 \sin 2\theta \).

5.[20pt.]  
(a) Use L'Hôpital's rule to help evaluate the improper integral:

\[
\int_1^\infty \frac{\ln x}{x^2} \, dx.
\]

(b) Make the \( u \)-substitution \( u = \sqrt{x} \) in the improper integral:

\[
\int_1^\infty \frac{dx}{\sqrt{x}(x + 4)},
\]

and evaluate the resulting definite integral.

6.[20pt.] Use partial fraction decompositions to evaluate the following integrals:

(a) \( \int \frac{2x^2 + 3x + 3}{(x + 1)^3} \, dx \).
(b) \( \int \frac{x^5 - 4x^3 + 1}{x^3 - 4x} \, dx \).

7.[20pt.]  
(a) Find the volume of the solid that results when the region above the \( x \)-axis and below the graph of the ellipse:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; (a > 0, \; b > 0),
\]
is revolved about the \( x \)-axis.

(b) Let \( a_n \) be the average value of the function \( f(x) = \frac{1}{x} \) over the interval \([1, n]\). Determine whether the sequence \( \{a_n\} \) converges and if so find its limit.

8.[20pt.] Evaluate the integral: \( I(x) = \int \frac{x^3}{\sqrt{x^2 + 3}} \, dx \) by:

(a) using Integration by parts.
(b) the substitution \( u = \sqrt{x^2 + 3} \).

9.[20pt.]  
(a) Find the radius of convergence and the interval of convergence for the power series:

\[
\sum_{k=0}^{\infty} \frac{(2x - 3)^k}{4^{2k}}.
\]
(b) Differentiate the Maclaurin series:
\[
\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \text{ for } -1 < x < 1
\]
and use the result to show that: \[\sum_{k=1}^{\infty} kx^k = \frac{x}{(1-x)^2}, \text{ for } -1 < x < 1.\]

10.[20pt.]
(a) The region bounded by the x-axis, the graphs of the equation \(y = x^2 + 1\), the lines \(x = -1\) and \(x = 1\) is revolved about the x-axis. Use the the Disk Method to find the volume of the resulting solid.
(b) Use Integration by parts to evaluate: \(\int_0^1 \tan^{-1} x \, dx\).

11.[20pt.]
(a) For the convergent infinite series: \(\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}\), how many terms \(n\) are needed to guarantee that the partial sum \(S_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^4}\) is within \(1 \times 10^{-10}\) of the actual sum \(S = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}\)?
[Hint: Use the error estimate \(|S - S_n| \leq a_{n+1}\), where the \(n\)th term \(a_n = \frac{1}{k^4}\).]
(b) Evaluate the integral: \(\int_0^{\pi/2} \frac{4 \sin x}{1+\cos^2 x} \, dx\).

12.[20pt.]
(a) Find the volume of the solid whose base is the region bounded by the curves \(y = \sqrt{x}\) and \(y = \frac{1}{\sqrt{x}}\), for \(1 \leq x \leq 4\) and whose cross-sections perpendicular to the x-axis are squares.
(b) Use a suitable substitution to show that: \(\int_0^1 \frac{x}{x^4+1} \, dx = \frac{\pi}{8}\).

13.[20pt.]
(a) Show that \(k^k \geq k!\).
(b) Use the comparison test to show that the series \(\sum_{k=1}^{\infty} \frac{1}{k^k}\) converges.
(c) Use the root test to show that the series \(\sum_{k=1}^{\infty} \frac{1}{k^k}\) converges.

14.[20pt.] Suppose that the sequence \(\{a_k\}\) is defined recursively by:
\[
a_0 = c, \quad a_{k+1} = \sqrt{a_k}.
\]
Assuming that the sequence converges, find its limit if:
(a) \(c = \frac{1}{2}\), \(b)\ c = \frac{3}{2}\).

15.[20pt.] A ship is at anchor in 80 ft of water. The anchor weighs 500 lb, and the chain weighs 20 lb/ft. The anchor is to be pulled up as the ship gets under way.
(a) How much force must be exerted to lift the anchor as it comes aboard the ship? Write an equation expressing the force in terms of the displacement \(y\) of the anchor from the ocean floor.
(b) How much work must be done to raise the anchor 80 ft from the ocean floor to the point where it comes aboard the ship?