

DEPARTMENT OF MATHEMATICS HOWARD UNIVERSITY
CALCULUS II (Math-157)- FINAL EXAMINATION
May 2, 2007

ANSWER ANY TEN (10) QUESTIONS. SHOW ALL WORK.
The time for this examination is two (2) hours.

1. [20 pts] (a) Sketch and find the area enclosed by $r = 2 \sin 2\theta$.

(b) Find the slope of the tangent line to $r = 2 \cos 3\theta$ at $\theta = \pi/6$.

2. [20 pts] (a) Find the arc length of the parametric curve:

$$x = e^t(\sin t + \cos t), \quad y = e^t(\cos t - \sin t), \quad (1 \leq t \leq 4).$$

(b) Find the area of the surface of revolution generated by revolving the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, about the x-axis.

3. [20 pts] Find the volume of the solid generated by revolving the region in the first quadrant that is above the parabola $y = x^2$ and below the parabola $y = 2 - x^2$ about the y-axis.

4. [20 pts] Find the radius of convergence and the interval of convergence of the series: $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{3^k(k+1)}$.

5. [20 pts] Decide whether the following series converge or diverge. Justify your answer in each case and state the test that you are using.

$$(i) \sum_{k=3}^{\infty} \frac{(2k)!}{4^k} \qquad (ii) \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

6. [20 pts] Determine whether the following series converge absolutely, converge conditionally, or diverge, giving reasons for your answers.

$$(i) \sum_{k=2}^{\infty} (-1)^k \frac{k}{\ln k} \qquad (ii) \sum_{k=1}^{\infty} \frac{\sin k}{k^3} \qquad (iii) \sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k}$$

7. [20 pts] (a) Find the Maclaurin series for $f(x) = 1 + 2007x - x^2 + x^3$.

(b) Find the first four Taylor polynomials for $\ln x$ about $x = 2$.

(c) Find the Taylor series for $f(x) = 1 + 2x - x^2 + x^3$ around $x = 1$.

8. [20 pts] Use integration by parts to evaluate the integral

$$I(x) = \int \frac{x^3}{\sqrt{x^2 + 3}} dx.$$

9. [20 pts] Evaluate the integral

$$\int \frac{\cos \theta}{\sin^2 \theta + 4 \sin \theta - 5} d\theta$$

10. [20 pts] Evaluate each of the following integrals

(a) $\int \frac{x^3 - 2x^2 + 2x - 2}{x^2 + 1} dx$

(b) $\int e^x \sqrt{1 - e^{2x}} dx$

(c) $\int_0^{\frac{\pi}{8}} \sec^3 2x \tan 2x dx$

11. [20 pts] Decide whether the following integrals converge or diverge. Justify your answer in each case.

(a) $\int_1^{\infty} \frac{(\ln x)^2}{x} dx$

(b) $\int_0^{\infty} \frac{1}{1 + 9x^2} dx$

12. [20 pts] A cone-shaped water reservoir is 20 ft in diameter across the top and 15 ft deep. If the reservoir is filled to depth of 10 ft, how much work is required to pump all the water to the top of the reservoir?