Final Exam
015-157 Calculus II
Spring, 1999

Please do all problems. Points are written to the left of each problem.

50 pts 1. Evaluate the following integrals:
   (a) \( \int \frac{3x}{1 + x^2} \, dx \)
   (b) \( \int \frac{1}{x(\ln x)^2} \, dx \)
   (c) \( \int xe^{-2x} \, dx \)
   (d) \( \int \ln \sqrt{x} \, dx \)
   (e) \( \int \frac{1}{(x + 1)(x - 2)} \, dx \)

10 pts 2. Approximate the integral \( \int_{1}^{2} \frac{1}{x} \, dx \) by one of the following methods, with \( n = 4 \): \( L_4 \), the left endpoint sum; \( R_4 \), the right endpoint sum; \( M_4 \), the midpoint rule; or \( T_4 \), the trapezoid rule.

10 pts 3. Determine whether the improper integral \( \int_{0}^{4} \frac{dx}{\sqrt{x}} \) converges or diverges. If it converges, find its value.

5 pts 4(a) Find the area bounded by the graphs of \( y = x \) and \( y = 4x - x^2 \).

10 pts 4(b) Find the volume generated when the region bounded by \( y = x \) and \( y = 4x - x^2 \) is rotated around the x-axis.

10 pts 5. Find the length of the curve parametrized by \( x(t) = t^2 \) and \( y(t) = \frac{3}{2}t^3 \) for \( 0 \leq t \leq 1 \).

10 pts 6. (a) Find an equation in \( x \) and \( y \) that has the same graph as the polar equation \( r = a \sin \theta \) with \( a > 0 \).

   (b) Sketch the graph.

10 pts 7. Find the area of the region bounded by the cardioid \( r = 2 + 2 \cos \theta \).
10 pts  8. Solve the initial value problem.

\[ \frac{dy}{dx} = \frac{1 + x}{xy}, \quad x > 0, \quad y(1) = 4. \]

10 pts  9. A bacterial culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours, there are 8,000 bacteria.
(a) Find an expression for the number of bacteria after \( t \) hours.
(b) When will there be 30,000 bacteria?

30 pts  10. Tell whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(a) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{3}{n + 4} \]

(b) \[ \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} \]

(c) \[ \sum_{n=1}^{\infty} \frac{(n + 1)5^n}{n3^{2n}} \]

20 pts  11. Find the radius of convergence and the interval of convergence of the following series.

(a) \[ \sum_{n=0}^{\infty} \frac{x^n}{n + 2} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(x - 4)^n}{n5^n} \]

10 pts  12(a) Find the McLaurin polynomial \( P_3(x) \) for \( f(x) = e^x \). (That is, the Taylor polynomial with \( a = 0 \) and \( n = 3 \).)

5 pts  12(b) Estimate the accuracy of the polynomial on \( 0 \leq x \leq 0.1 \) using the remainder term.