Final Exam

Math 158 Calculus III

Fall 2002

Please do 14 of the following problems. Be sure to indicate which 14 problems you want to have graded.

1. Find parametric equations for the line through the point $(0, 4, 0)$ and perpendicular to the plane $4x - 5y + 6z + 2 = 0$.

2. Determine whether the points $(1, 2, -1)$, $(3, 3, -4)$, $(2, 2, 1)$, and $(5, 3, 0)$ lie in the same plane. If they do, find an equation of that plane.

3. Describe the surface $z = 3x^2 + 3y^2$ and write equations for this surface in spherical coordinates and in cylindrical coordinates.

4. The position vector of a moving particle at time t is $\vec{r}(t) = (2, t^3, -16 \ln t)$. Find the speed, velocity, and acceleration of the particle at time $t = 1$.

5. Let $\vec{r}(t) = (3t^3 + 1, 2t^{9/2})$ and let $C = \{\vec{r}(t) : 0 \le t \le 1\}$. Find the length of the curve C.

6. Let $f(x, y) = x^2 y + y \ln(xy + 2)$. Find the directional derivative of f at the point $(1, -1)$ in the direction toward the point $(5, 2)$. That is, find $f_{\vec{u}}(1, -1)$ where $\vec{u}$ is the unit vector parallel to the vector from $(1, -1)$ to $(5, 2)$.

7. Find an equation of the plane tangent to the graph of $x^2 y + xyz^2 + z = 1$ at the point $(0, 2, 1)$.

8. Find the local extrema of the function $f(x, y) = x^4 + y^4 + 4xy$.

9. Using Lagrange multipliers, find the maximum and minimum values of the function $f(x, y) = x - 2y + 1$ on the ellipse $x^2 + 3y^2 = 21$.

10. Evaluate $\displaystyle\iint_R y^2 \cos x \, dA$ where R is the region bounded by the curve $x = y^3$ and the lines $y = -1$, $y = 1$, and $x = 3$.

11. Evaluate $\displaystyle\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} (1 + x^2 + y^2) \, dx \, dy$. (You may find it helpful to convert to polar coordinates.)

12. Determine the surface area of the portion of the plane $2x - 3y + z = 5$ for which $0 \le x \le 2$ and $3 \le y \le 6$.

13. Find the centroid of the region bounded by the graphs of $y = x^4$ and $y = 1$. (You should not have to do any computation to determine the value of $\overline{x}$.)

14. Let $\vec{F}(x, y) = (4y + 2x, 4x + \cos y)$. Find a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $\vec{\nabla} f(x, y) = \vec{F}(x, y)$ and $f(1, 0) = 2$.

15. State Green's Theorem and verify Green's Theorem for the vector field $\vec{F}(x, y) = 2xy\vec{\imath} + 3y^2\vec{\jmath}$ and the region R bounded by the graphs of $y = 1 - x^2$ and $y = 0$.

16. Let $\vec{r}(u, v) = (u + v, u, u - v)$ and let $\sigma = \{\vec{r}(u, v) : 0 \le u \le 1 \text{ and } 0 \le v \le u\}$. Evaluate the surface integral $\displaystyle\iint_{\sigma} xy^2 \, dS$.

17. Following are two polar coordinate equations. Identify the graph of each, give the eccentricity and the distance from the focus to the directrix.

(a) $r = \dfrac{2}{1 + 2\cos\theta}$

(b) $r = \dfrac{2}{3 - \sin\theta}$