Final Exam

Math 158 Calculus III

Fall 2003

Please do 10 of the following problems. (Be sure to indicate which ten problems you want to have graded.)

1. Find an angle of rotation that eliminates the xy-term from the equation

$$11x^2 + 4\sqrt{3}\,xy + 7y^2 - 1 = 0\,.$$

Rewrite the equation in terms of the new coordinate system and sketch the curve displaying both coordinate systems.

2. Let $L_1 = \{\langle 3,1,5\rangle + t\langle 1,-1,2\rangle : t \in \mathbb{R}\}$ and let $L_2 = \{\langle 1,4,2\rangle + u\langle 0,1,1\rangle : u \in \mathbb{R}\}$.

(a) Find the point at which the lines L_1 and L_2 intersect.

(b) Find the cosine of the acute angle between the lines L_1 and L_2 at the point of intersection.

3. The planes $x - y + z = 1$ and $2x + y + 3z = 5$ intersect in a line.

(a) Find parametric equations for the line of intersection.

(b) Find an equation for the plane that contains the line of intersection and passes through the point $(1, 0, -3)$.

4. Let $z = \sqrt{x^2 + y^2}$. Sketch the level curves $z = k$ for $k = 0$, $k = 1$, and $k = 3$.

5. Let $f(x,y) = \dfrac{y^2}{x^2 + y^2}$. Show that $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

6. Let $f(x,y,z) = x^3y^2z + \sin(xy)$. Find f_x, f_y, and f_z.

7. Given that z is a function of x and y satisfying the equation $x^2y + y^2z + z^2x = 1$, use implicit differentiation to compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

8. Let $f(x,y) = xe^y - ye^x$. Find the directional derivative of f at the point $(1,0)$ in the direction of $3\vec{\imath} + 4\vec{\jmath}$.

9. Let $f(x,y) = 5 + 4x - 2x^2 + 3y - y^2$. Find the absolute maximum and minimum of f on the region bounded by the lines $y = 2$, $y = x$, and $y = -x$.

10. Find the points on the graph of $2xy^2 = 1$ that are closest to the origin.

11. Evaluate $\int_1^2 \int_x^{2x} xy \, dy \, dx$.

12. Find the centroid $(\overline{x}, \overline{y})$ of the triangular region with vertices $(0,0)$, $(2,0)$, and $(1,2)$. (You shouldn't need to use calculus to find $\overline{x}$.)

13. Evaluate $\int_{-2}^0 \int_0^{\sqrt{4-x^2}} \int_0^{x^2+y^2} 2z \, dz \, dy \, dx$.

14. Evaluate $\iint_R x \, dA$ where R is the region bounded by the lines $x = 2y$, $x = 2y + 3$, $x = y$, and $x = y + 3$. (You will probably find Jacobians to be helpful.)

15. Let $\vec{r}(t) = (\cos t, \sin t)$, let $\vec{F}(x, y) = (2xy + 1)\vec{\imath} + (\cos(\pi y) + x^2)\vec{\jmath}$, and let $C = \{\vec{r}(t) : 0 \leq t \leq \frac{\pi}{2}\}$. Compute $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$. (Find a function ϕ such that $\vec{F} = \vec{\nabla}\phi$.)