Enough work must be given to justify your solutions. Each solution (1-20) is valued at 10 points.

I. Write an equation for each of the following:
1. The line passing through the point $P(1, -1, 5)$ with direction vector $i - j + k$.
2. The plane passing through the point $P(1, -1, 5)$ with normal vector $i - j + k$.
3. The plane through the points $P(1, 1, 1)$, $Q(-1, 3, 2)$ and $S(1, -1, 2)$.
4. The plane passing through the point $P(1, 1, 1)$ and containing the line of intersection of the planes with equations $x - y + z = 5$ and $2x - y + z = 10$.
5. The line normal at $P(0, 0, 0)$ to the surface $S$ defined by the equation $e^z \sin y + e^y \sin z + e^z \sin x = 0$.
6. The plane tangent at $P(0, 0, 0)$ to the surface $S$ defined by the equation $e^z \sin y + e^y \sin z + e^z \sin x = 0$.

II. Do each of the following problems:
7. Give a sketch for the surface with equation $z = x^2 + y^2$. Identify the traces in the coordinate planes and 3 distinct level curves.
8. Let $C$ be the plane curve defined by $r(t) = e^{2t}i + e^{4t}j$.
   (a) Give a sketch for $C$.
   (b) Find and sketch $r(0)$.
   (c) Find $r'(0)$ and sketch $r'(0)$ at the point on $C$ where $t = 0$.
9. If the curve $C$ is defined parametrically by $x = t - t^2$, $y = 1 - t^3$, find the curvature of $C$ at the point $P(0, 1)$.
10. Give a definite integral for the length of the curve $C$ defined by
    $r(t) = 4 \cos ti + 9 \sin tj + tk$ from $t = \pi/2$ to $t = \pi$ (DO NOT INTEGRATE).
11. If $r(t) = e^t(\cos ti + \sin tj + k)$ is the position vector for a moving point at time $t$, find the velocity, acceleration, and speed of the point at $t = \pi/2$.

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12. Find the distance from the plane with equation $3x - y + 2z = -1$ to the point $P(-1, 2, 1)$.

In Problems 13 and 14 let

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

13. Show that $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

14. Find the directional derivative of $f$ at $(0, 0)$ in the direction of the vector $4\mathbf{i} + 3\mathbf{j}$ if it exists. If it does not exist, explain why.

15. Find every point on the surface $S$ defined by $z = 3x^2 + 12x + 4y^3 - 6y^2 + 5$ where the tangent plane to $S$ at the point is horizontal.

16. Find and classify the critical points of $f$ defined by $f(x, y) = 3x^2 + 6xy + 2y^3 + 12x - 24y$.

17. The function $f$ defined on the part of the surface $xyz = 8$ where $x, y$ and $z$ are positive by $f(x, y, z) = xy + xz + yz$ has a minimum value. Use the method of Lagrange Multipliers to find the minimum value.

18. Find the maximum directional derivative for $f$ defined by $f(x, y, z) = e^{x-y-z}$ at $P(5, 2, 3)$ and the direction in which this derivative occurs.

19. Integrate:

(a) $\int_{-1}^{2} \int_{1}^{2} (12xy^2 - 8x^3)dy
dx$  
(b) $\int_{0}^{1} \int_{2x}^{2} e^{y^2}dy
dx$

(c) $\int \int_{R}(4x - y)dA$ where $R$ is the region bounded by the graphs of $x = y^2$ and $x = 2y$.

20. Let $A(a, b)$ represent the area of the region bounded by the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) Set up an iterated integral to find $A(a, b)$.

(b) Integrate the iterated integral in part (a) to find $A(a, b)$.

(c) What is $A(a, a)$? Interpret.