Please provide step by step solutions with explanations for each step.
Total 200 points; Time limit 2 hrs.

Part I: 15 points each, do all the problems

1. Determine whether the line whose equation is given by \( \mathbf{r}(t) = \mathbf{i} + t(2\mathbf{i} + \mathbf{k}) \) is parallel or perpendicular or neither to the plane given by \( x + 3y - 2z = 5 \).

2. A particle is travelling along a curve whose parametric equation is given by \((1+3t^2)\mathbf{i} + 4t^2\mathbf{j} + 2\mathbf{k}\) where \( t \) is time in seconds. Find the distance travelled (arc-length) from \( t = 0 \) to \( t = 2 \), and the velocity and speed at \( t = 2 \) seconds.

3. Find all first and second partial derivatives of \( x^3y^2 - 2\cos(xy) \).

4. Find all relative extrema and saddle points of \( f(x, y) = x^3 + 3xy + 6y \).

5. From the equation \( xe^x + yz = x^2 \) that determines \( z \) as a function of \( x \) and \( y \) implicitly, find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) using implicit differentiation.

6. Find the equation of the tangent plane and normal line to the surface \( z = 2xy - y^2 \) at the point \((1,2,0)\).

7. Evaluate \( \int_0^1 \int_{\sqrt{x}}^2 \sin(\pi y^3) \, dy \, dx \) by first changing the order of integration.

8. Evaluate \( \int_0^1 \int_{1+x}^{x+z} x \, dy \, dz \, dx \)

9. Find the area of the portion of the surface \( z = xy \) inside the cylinder \( x^2 + y^2 = 1 \).

10. Use the transformation \( u = x/3, v = y/4 \) to evaluate the area inside the ellipse \( \frac{x^2}{9} + \frac{y^2}{16} = 1 \) by integrating over the disk \( u^2 + v^2 \leq 1 \).

Part II. Answer any two. Each carries 25 points

1. (a) (15 points) Find the work done by the force \( \mathbf{F} \) given by \( \mathbf{F}(x, y) = y^2\mathbf{i} - 2x^2\mathbf{j} \) acting on a particle that moves along the circle \( x = \cos t, y = \sin t \) from \((1,0)\) to \((0,1)\). [You may use: \( \int_0^{\pi/2} \cos^2 t = \int_0^{\pi/2} \sin^3 t = 2/3 \).]

   (b) (10 points) Find the divergence and the curl of the vector field \( \mathbf{F} \) given by \( \mathbf{F}(x, y, z) = \cos x\mathbf{i} + \sin x\mathbf{j} + \ln(xy)\mathbf{k} \).

2. Find the volume of the solid enclosed by the cylinder \( x^2 + y^2 = 4 \) and the surface \( z^2 - x^2 - y^2 = 4 \).

3. Find the point on the plane \( x+3y+z = 1 \) within the first octant \((x \geq 0, y \geq 0, z \geq 0)\) that is closest to the origin.

4. Suppose a particle is moving with constant velocity, i.e., \( \mathbf{r}'(t) = \mathbf{c} \) for some fixed vector \( \mathbf{c} \). Let the magnitude of \( \mathbf{c} = c \). Show that \( \frac{d^2}{dt^2} ||\mathbf{r}(t)||^2 = \frac{d^2}{dt^2} (\mathbf{r}(t) \cdot \mathbf{r}(t)) = 2c^2 \) and \( \frac{d}{dt} (\mathbf{r}(t) \times \mathbf{r}'(t)) = 0 \).