Final Exam
Math 159 Calculus III
Spring 2006

Choose only 8 questions from #1 - #12 and only 7 questions from #13 - #21.

[*20 pts*] 1. Find the maximum and minimum values of the radius of curvature ρ for the curve $x = \cos t$, $y = \sin t$, $z = \sin t$; $0 \le t < 2\pi$.
(Note: $\rho(t) = 1/\kappa(t)$ where κ denotes the curvature.)

[*20 pts*] 2. Evaluate the double integral

$$\iint_R \sqrt{x^2+y^2}\, dA, \quad \text{where } R = \{\,(x,y) : x^2+y^2 \le 1\,\}.$$

[*20 pts*] 3. Evaluate the double integral

$$\iint_R \frac{xy}{\sqrt{1+x^2+y^2}}\, dA, \quad \text{where } R = \{\,(x,y) : 0 \le x \le 1, 0 \le y \le 1\,\}.$$

[*20 pts*] 4. Find the surface area of the surface $z = 1 - x^2 - y^2$ with $1 - x^2 - y^2 \ge 0$.

[*20 pts*] 5. Evaluate the iterated integral $\displaystyle\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}} zr\, dz\, dr\, d\theta$.

[*20 pts*] 6. Use the transformation $u = x - 2y$, $v = 2x + y$ to find

$$\iint_R \frac{x-2y}{2x+y}\, dA,$$

where R is the rectangular region enclosed by the lines
$x - 2y = 1$, $x - 2y = 4$, $2x + y = 1$, $2x + y = 3$.

[*20 pts*] 7. Use Lagrange multipliers to find the maximum and the minimum values of $f(x,y,z) = xyz$ subject to the condition $x^2 + y^2 + z^2 = 1$.

[*20 pts*] 8. Find the local maxima, minima, and saddle points of $f(x,y) = e^{-(x^2+y^2+2x)}$.

[*20 pts*] 9. Find the equation of the tangent plane of $z = \ln\left(\sqrt{x^2+y^2}\right)$ at $(-1,0,0)$.

[*20 pts*] 10. Find the unit tangent and unit normal vectors to the graph of the curve $\mathbf{r}(t) = \ln t\,\mathbf{i} + t\,\mathbf{j}$ at $P(0,1)$. Sketch the curve showing the point of tangency. (Be careful in drawing the direction of the unit tangent and the unit normal vectors.)

[*20 pts*] 11. Find the unit vector in the direction in which $f(x,y,z) = \tan^{-1}\left(\dfrac{x}{y+z}\right)$ increases most rapidly at $(4,2,2)$.
(Note: $\dfrac{d}{du}(\tan^{-1} u) = \dfrac{1}{1+u^2}$)

[*6 pts*] 12. (a) Express the vector $\mathbf{v} = \langle -1, 4, 8\rangle$ as the sum of two orthogonal vectors such that one of them is parallel to $\mathbf{b} = \langle 2, -2, -1\rangle$. Use your decomposition to compute the distance from the point $(-1, 4, 8)$ to the line determined by the vector $\mathbf{b}$.

[*6 pts*] (b) Show that in 3-space the distance d from a point P to the line L that is passing through the points A and B can be given by the formula

$$d = \frac{\| \overrightarrow{AP} \times \overrightarrow{AB} \|}{\| \overrightarrow{AB} \|}$$

[*6 pts*] 13. Find the directional derivative of $f(s, y, z) = \sin(xyz)$ at $(1/2, 1/2, \pi)$ in the direction of $\langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\rangle$.

[*6 pts*] 14. Find f_x for $f(x, y) = \int_1^{xy} e^{t^2}\, dt$.

[*6 pts*] 15. (a) Find parametric equations of the directed line *segment* from the point $P(2, 1, 3)$ to the point $Q(1, 3, -2)$.

[*6 pts*] (b) Use vectors to determine whether the points $P_1(3, 1, 3)$ $P_2(1, 5, -1)$ and $P_3(4, -1, 5)$ are collinear.

[*6 pts*] 16. Let $\mathbf{r} = \langle x, y\rangle$, and fix two distinct points $\mathbf{r}_1 = \langle x_1, y_1\rangle$ and $\mathbf{r}_2 = \langle x_2, y_2\rangle$. Given $a > 0$ and $\|\mathbf{r_2} - \mathbf{r}_1\| > a$, <u>describe</u> the set of all points (x, y) for which $\|\mathbf{r} - \mathbf{r}_2\| - \|\mathbf{r} - \mathbf{r}_1\| = a$. (Do not attempt to derive the equation in standard form algebraically.)

[*6 pts*] 17. (a) Let A, B and C be three distinct noncollinear points in 3-space. Describe the set of points P that satisfy the vector equation $\overrightarrow{AP} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = 0$.

[*6 pts*] (b) Determine whether the points $A(0, 0, 0)$, $B(1, -1, 1)$, $C(2, 1, -2)$ and $D(-1, 2, -1)$ are coplanar, (i.e., lie on the same plane).

[*6 pts*] 18. Determine whether the line L_1: $x = 3 - t$, $y = 5 + 3t$, $z = -1 - 4t$, and the line L_2: $x = 8 + 2t$, $y = -6 - 4t$, $z = 5 + 2t$ have a point of intersection.

[*6 pts*] 19. Find the parametric equations of the line through $(2, 0, -3)$ that is parallel to the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $x - y - z - 5 = 0$.

[*6 pts*] 20. Find parametric equations for $\mathbf{r} = \langle 2 + \cos 3t, 3 - \sin 3t, 4t\rangle$, $0 \le t \le \pi/4$, using arc length s as a parameter. Take the point on the curve where $t = 0$ as the reference point.

[*6 pts*] 21. Find the work done by a force field $\mathbf{F}(x, y) = \langle x^3 - y^3, xy^2\rangle$ along the curve C parametrized by $x = t^2$, $y = t^3$, $-1 \le t \le 0$.