5/7/03 180-01 Linear Algebra Final Exam Spring 03 Sitaraman

Instructions

1. Please provide step by step solutions with explanations for each step. **Otherwise full credit will not be given.**
2. Each problem is worth 20 points unless otherwise specified. Total 200 points.
3. Time limit 2 hrs.

1. Check whether the following system is consistent, and if so, does it have a unique solution:

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & 0 & 1 & 1 \\ 5 & 2 & 5 & 6 \end{bmatrix}$$

2. $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are three vectors in $\mathbf{R}^4$. Write down five vectors that are in the subspace of $\mathbf{R}^4$ spanned by these three vectors. Find three vectors other than $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ that will form a basis to this subspace spanned by them.

3. In a certain town, 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women. Assuming that the total population of women remains constant, how many married women and how many single women will be there after 1 year? After 2 years? You must write down the matrix equation and use it to find the answers.

The next few problems involve the matrix A that is given by

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 5 & 8 \\ 4 & 2 & 6 \end{bmatrix}.$$

4. Find $det A, A^{-1}$, and $det(A^{-1})$.

5. What is the null space of A? If T is the linear transformation on $\mathbf{R}^3$ associated to A, what is its nullity? Is T one-one? Give reasons for your answer.

6. Find a basis for the column space of A and the rank of T.

7. Find the images of the following vectors under T: [1,1,1], [2,-5, 7]. Is [1,1,1] in the image space of T?

8. Say whether the following are true or false, with reasons for your answers (i.e, give proof if you think it is true, and give a counter-example if you think it is false):

(a) Any linear transformation S on any vector space sends the zero vector to the zero vector.

(b) The null space of a linear transformation is a subspace of its image space.

(c) The eigenspace of a linear transformation corresponding to any eigenvalue is a subspace of its null space.

(d) If S is a linear transformation, S sends vectors from a vector space V to vectors in V itself, and S is one-one, then S is also onto (i.e, the image space of S is also all of V).

9. Given that $det(A) = 2, det(B) = 3, det(C) = 5$, find the determinants of the following (A, B, C all have the same number of rows and columns): AB, B^2, BCA, C^{-1}. [Hint: use the fact that the determinant of a product of matrices is the same as the product of the determinants]

10. Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$.

Bonus problem (10 points). X has \$100 to spend on spring break. Beer costs \$3 for a bottle, Food costs \$8 for a meal, and the ride to the beach costs \$16. The number of beers purchased is twice the number of meals. How many times can X eat, how many beers can X drink, and how often can X go to the beach?