There are 8 problems, with the point value of each problem adjacent to it. Show all work neatly and clear answers. All explanations must be coherent and written in college level English.

1.(60) Find the general solution of each differential equation:

a) $x^2y' + 2xy = 6y^4$ b) $y' = \sin(x + y + 3)$

c) $2y'' - 3y' - 5y = 10t^2$ d) $(\sin^2 x - y)dx - \tan x dy = 0$

2.(30) Find the solution to each initial value problem

a) $\frac{d^2y}{dt^2} + 4y = 3\cos(2t), \qquad y(0) = y'(0) = 0$

b) $2x^2ydy + 2xy^2dx - 2ydy + dx = 0 \qquad y(0) =$

3.(30) Use Laplace transforms to solve the initial value problems:

a) $y^{(4)} - y = 3; \qquad y(0) = y'(0) = y''(0) = y'''(0) = 0$

b) $\frac{dy}{dt} + 7y = u_2(t), \qquad y(0) = 3$

4.(30) Consider the second-order equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$$

a) Convert the equation to a first-order linear system
b) Compute the eigenvalues of the system
c) For each eigenvalue, pick an associated eigenvector $\mathbf{v}$,
d) Determine the general solution $\mathbf{Y}(t)$ for the system.

5.(10) Give an example of a differential equation that models an overdamped harmonic oscillator.

6.(15) Match each of the following systems with one of the direction fields on the reverse. Give a clear justification f

I

$$\frac{dx}{dt} = x + 4y \qquad \frac{dy}{dt} = -3x + 2y$$

II

$$\frac{dx}{dt} = -2x - 2y \qquad \frac{dy}{dt} = -x - 3y$$

$$\frac{dx}{dt} = \qquad \frac{dy}{dt}$$

7.(15) Locate the bifurcation values for the one-parameter family shown and draw the phase lines for the values of t equal to the bifurcation value.

$$\frac{dy}{dt} = y^2 + 3y + \alpha$$

8.(15) A 10 gallon bucket is initially full of pure water. We begin dumping salt into the bucket at a rate of 1/4 pour minute is drained from the bucket. We add pure water to keep the bucket full and constantly stir the solutic salt in the bucket after

a) 2 minutes b) several hours

(over)