Gravity induced dispersion for nearly flat vortex sheets

Using techniques from the theory of oscillatory integrals, we prove rigorous estimates which show that the linearization of the vortex sheet equations of motion about a quiescent state disperse under certain circumstances. Such dispersion is only possible only through the joint effects of surface tension (which damps high frequency modes) and gravitation (which damps low frequency modes). This work is joint with D. Spirn.

An experimental study of the correlation between wave nonlinearity and low-frequency waves in shoaling water

The correlation between the nonlinearity of random gravity (primary) waves and the low-frequency waves induced by the primary waves on inclined beaches were studied experimentally in a wave flume at the Physical Oceanography Laboratory, Ocean University of China. Three plane beaches with different slopes (1/20, 1/30 and 1/40) were used in separate experiments. Incident primary waves of the Person-Moskowitz (PM) spectrum were mechanically generated with varying significant wave height from 0.047 to 0.125 m. The time series of wave surface elevations at various water depths along the beach were simultaneously recorded. Our results show that the probability distribution of surface elevations gradually deviates from the Gaussian function as the water depth decreases. The skewness and kurtosis of the wave surface elevations, that represent the nonlinearity of primary waves are functions of the nondimensional parameter $H_s/d$, where $H_s$ is the local significant wave height and $d$ is the water depth. The energy ratios between low-frequency and primary waves strongly correlate to the local surface skewness and the growth (dissipation) rate of low-frequency waves on beaches is controlled by the Iribarren number, $\xi = \beta / (H_0/L_0)^{1/2}$, where $\beta$ is the beach slope and $H_0$ and $L_0$ are the significant wave height and wavelength of incident primary waves, respectively.
May 14, 2008: Wednesday

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Modeling Water Waves with Particles

Numerical modeling of water waves has been underway for four decades and our abilities have advanced from steady nonlinear wave solutions to plunging breakers. This talk briefly discusses how we got to the present and then focuses on using smoothed particle hydrodynamics (SPH) to study waves.

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Wave dynamics in electrified viscous film flows

Falling film flows are found in numerous applications including coating flows and heat or mass transfer systems. In some applications we may want to suppress wave activity or pronounced wave features (coating flows) and in others promote it (heat or mass transfer). In this talk I will show how these two effects can be achieved by applying electric fields. In the first paradigm, we consider highly viscous film flows on flat substrates which are inherently stable and show that instability and complex nonlinear dynamics including chaotic behavior, can be induced by a sufficiently strong electric field. This is done by deriving asymptotically a novel Kuramoto-Sivashinsky type equation that contains nonlocal energy supplying terms. In the second example, we show how electric fields suppress wavy ridges which are found in coating flows over topographically structured substrates. This is done numerically and analytically using asymptotic methods. We close by presenting some direct numerical simulations which indicate that electric fields may be used to enhance mixing in microfluidic devices.

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A Numerical Method for a Maximum Entropy Problem

The flow of a fluid as described by the 2-d Euler equations is
extremely complex and difficult to simulate numerically. A model for the equilibrium (most probable) solution to the Euler equations was derived using statistical mechanics. The solution requires the maximization of a nonlinear functional subject to the conserved quantities of the flow. We will present this model along with a numerical method to find solutions to the maximization problem. The model predicts such dynamical phenomena as the coalescence of vortex patches and roll-up of vortex sheets.

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Wave Modulated Turbulent Fields at the Ocean Surface and Related Air-Sea Fluxes

Ocean surface processes and air-interaction in general, have recently received increased attention and it is now accepted that small-scale surface phenomena such as surface waves, turbulence, bubbles and droplets, can play a crucial role in the air-sea fluxes of heat, mass and momentum, with important implications for weather and climate studies. Yet, despite good progress in recent years, the air-sea interface and the adjacent atmospheric and marine boundary layers have proven to be difficult to measure in all but the most benign conditions. This is in part because, as opposed to the flow over a solid flat boundary, from which models of air-sea fluxes are derived, the ocean surface is subject to drift currents and populated with surface waves and turbulent eddies over a large spectrum of scales. The difficulty of making measurements is further complicated by the fact that there may be significant interactions between the currents, the surface waves, and the turbulence.

We present data from three field experiments from \textit{R/P FLIP} and Scripps pier where we have used novel optical and infrared techniques aimed at simultaneously studying multiple aspects of the air-sea interface. The data show that the skin layer temperature is modulated by the surface wave field. This is not a novel result, but we show here that this modulation leads to a wave supported air-sea heat flux. In addition, we show that the surface kinematic fields such as the divergence and vorticity are also coherent with the temperature field. This is the result of the presence of Langmuir circulations (or Langmuir turbulence) at the ocean surface which is in fact, one of the better understood interactions between vortical fields and the Stokes drift generated by the surface waves. Consequently, we show that the surface turbulence and kinematic fields are in turn modulated by the surface waves, just as the skin temperature is. For example, the vortex lines are stretched and
compressed by traveling surface waves leading to enhanced vertical vorticity at the crest of the waves. The turbulence is modulated by the waves in a fashion that is qualitatively consistent with rapid distortion theory whereby the shearing effect of the wave orbital motion is more important to that of turbulence self interaction. In summary, our results show that the surface waves modulate the thermal molecular layer as well as the turbulence at the surface of the ocean. These results will be discussed on the context of their influence on the modulation of air-sea fluxes.

May 15, 2008: Thursday

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Approximate Periodic Solutions for the Rapidly Rotating Shallow-Water Equations

We study the stabilizing effect of rotational forcing in the nonlinear setting of two-dimensional shallow-water equations. The pressure-less version of these equations admit global smooth solution for a large set of sub-critical initial configurations. But what happens with more realistic models, in the presence of pressure? it is shown that when rotational forcing dominates the pressure, it prolongs the life-span of such sub-critical solutions, for a time period dictated by the ratio \( \delta = \text{Rossby number}/\text{squared Froude number} \).

Our study reveals a "nearby" periodic-in-time approximate solution in the small delta regime, upon which hinges the long time existence of the exact smooth solution. These results are in agreement with the close-to periodic dynamics observed in the "near inertial oscillation" (NIO) regime which follows oceanic storms.

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Practical wind wave forecasting

The World Meteorological Organization (WMO) has classified wind waves as part of the weather, and hence wind wave forecasting represents a daily effort at many meteorological centers around the world. The lecture explores the history of wind wave forecasting, with a focus on the evolution of tools used in daily forecasting. Operational wave forecasting is associated with a large field of applied engineering and
research. A general overview will be given of this field, identifying main research areas, together with an overview of typical careers in this field.

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A semilinear wave equation with smooth data and no resonance having no continuous solution
(Joint work with J. F. Caicedo of Universidad Nacional de Colombia)

We prove that the hyperbolic problem
\begin{equation}
\begin{aligned}
\partial_{tt}(u) - \partial_{xx} + g(u) &= c \sin(x+t) & \quad x,t \in \mathbb{R} \\
u(x, t) &= u(x, t + 2\pi) = u(x+2\pi,t) & \quad x, t \in \mathbb{R},
\end{aligned}
\end{equation}

has no continuous solution when $|c|$ is large and
\begin{equation}
g(t) = \tau t + h(t) \quad \hbox{with} \quad \tau \in (0, \infty) - \{k^2 - j^2; k, j = 0, 1, \ldots\},
\end{equation}
and $h: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function with support in $[0, D]$ and such that
\begin{equation}h(D/2) < -\tau D/2.\end{equation}
We note that for some $t \in (0, D)$, $g'(t) < 0$, and that the problem has no resonance. This supplements our understanding of semilinear equations when the range of the derivative of the nonlinearity includes eigenvalues of infinite multiplicity.

Shu-Ming Sun

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Spectral stability of solitary waves on water of finite depth

The talk will discuss most recent development on stability theory of solitary waves in water using exact Euler equations. It is known that the exact equations have traveling solitary-wave solutions and it is still an open question whether the solitary-wave solutions are stable. Here, it will
be shown that the spectrum of a linear operator, which is obtained from the linearized Euler equations around the solitary-wave solution, is always on the left half (including the imaginary axis) of the complex plane. (This is a joint work with R. Pego.)

May 16, 2008: Friday

Harry Yeh
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Tsunami propagation from a finite source

Sea-bottom displacements associated with seismic events are usually confined to strips of large but finite aspect ratio. We analyze tsunamis that are initiated on such a strip and that propagate across a region of finite depth. We invoke the classical shallow-water-wave theory to obtain comprehensive descriptions of the non-dispersive aspects of the waves. The directivity of the energy radiation and the domain of pulse persistence are discussed. The pulse-persistent range together with the measured satellite data of the 2004 Indian Ocean Tsunami reveal that the initial water-surface displacements at the seismic source should be about 3 m on the average. This estimate is in good agreement with the solution obtained from the inversion of seismic waves.

Please note that this is an application of the solution recipe made by the late Professor George Carrier.

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Lagrangian Coherent Structures and Lagrangian Data Assimilation of Ocean Systems

Dynamical systems has an array of very versatile tools for the study of flow fields. In the first part of this talk I present results from studying flow fields of geophysical systems from a dynamical systems point of view. The systems that I study are the Rossby wave, a westward propagating planetary wave, and the Eastern Gulf of Mexico. More specifically I use a Lagrangian diagnostic to computationally distinguish the Lagrangian coherent structures, the equivalent invariant manifolds, that govern the field’s underlying dynamics. The Lagrangian diagnostic I use is the method of relative dispersion, a finite time method. As with all finite-time methods, a termination time for the relative
dispersion algorithm must be determined. A major result in this work is finding such a termination time with the aid of the statistics of the computed relative dispersion scalar.

In the second part of the talk, I introduce data assimilation, sequential data assimilation, the Kalman filter, and the extended Kalman filter. The flavor of extended Kalman filter that I use in this work directly assimilates Lagrangian tracer positions into the model. More specifically, I use this method to estimate the parameters of the streamfunction describing the flow field of a nonautonomous Rossby wave. In the first set of experiments, I set a benchmark by assimilating observations of one tracer launched in the flow, and discuss the factors affecting the results of the parameter estimation. In the second set of experiments, two passive tracers are launched in proximity in the flow field. With different initial conditions, the Lagrangian trajectories the two tracers follow are different, and the tracers can end up in different dynamical regions in the flow field. After making observations of the tracer positions, the two sets of observations are concurrently assimilated, and the parameters recovered from the two assimilations are compared; we find that the parameters are, in fact, comparable. The discrepancy in the two sets of parameter values recovered are dependent on the tracers’ launch locations, the time-dependency of the flow field, the frequency and accuracy of the observations, and the strength of the stochastic forcing of the true system.

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MATHEMATICAL METHODS FOR COUPLED NONLINEAR PDE'S

A coupled Nonlinear Schrödinger equation is studied. In particular, we study the special case of soliton and soliton like solutions. We show when exact soliton solutions exist and how they gave rise to initial value numerical results. Further, we explore approximate variational methods, that lead to more initial value numerical results. Finally, the consequences of the approximate variational results are explored in terms of its Local Lie structure.

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Study of two-dimensional wave patterns

In this talk, I shall first introduce the Boussinesq systems by comparing them with other water wave models and then present theoretical and numerical results on two-dimensional wave patterns. Some predictions on the qualitative behavior of the wave patterns will be made at the end of the talk.
Incipient Breaking Conditions for Short Wavelength Spilling Breakers

The crest profile histories of spilling breakers with wavelengths ranging from 0.1 to 1.2 m and generated by wind alone, by a mechanical wave maker alone, and by a mechanical wave maker in the presence of wind were studied experimentally. In all cases considered herein, the breaking process is initiated by the formation of a capillary-bulge pattern at the crest. It is found that the shapes of the capillary-bulge patterns at the point of incipient breaking are qualitatively similar for all wave generation conditions. However, the geometry of the patterns varies quantitatively with gravity wavelength and generation method. The data is analyzed in search of a self-similar crest shape that depends on a minimum number of simple length scales and physical principles. Some effects of surfactants are also explored.

Lake and Great-Lake Equations

Currents in lakes can be modeled by systems of shallow water equations that describe the long-time motion of an incompressible fluid contained in a shallow basin with a slowly spatially varying bottom, a free upper surface, under the influence of gravity, and in the limit of small characteristic velocities and very small surface amplitude. Gravity waves can be neglected in such regimes. We will present two such systems and discuss their mathematical structure.

Well-posedness of water waves

We will briefly review some of the different methodologies for well-posedness of waterwaves and the Euler equations. We will then describe a new method for treating free boundary problems in perfect fluids, and prove local-in-time well-posedness in Sobolev
spaces for the free-surface 3D Euler equations for arbitrary initial data, and without any irrotationality assumption on the fluid. This is a free boundary problem for the motion of a perfect liquid in vacuum, wherein the motion of the fluid interacts with the motion of the free-surface at highest-order. We will describe the geometry behind the a priori nonlinear estimates and the approximation schemes that must be developed in order to prove existence of solutions.

EXPERIMENTS

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All of the experiments on water waves were conducted in the afternoon sessions from 3:20 to 4:20 PM under the direction of Diane Henderson.

MODERATORS

Dan Williams, Department of Mathematics, Howard University; Sonia Smith, Department of Mechanical Engineering, Howard University; Joshua Leslie, Department of Mathematics, Howard University; Louise Raphael, Department of Mathematics, Howard University; M.F. Mahmood, Department of Mathematics, Howard University