1. [20 Points]
(a) Find the directional derivative of the function:
\[ f(x, y) = x^2 e^{-2y} \]
at the point \( P(2, 0) \) in the direction of the vector from \( P(2, 0) \) to \( Q(-3, 1) \).
(b) Find the maximum rate of increase of \( f(x, y) \) at \( P(2, 0) \).

2. [20 Points]
Use Chain Rule to find \( \frac{\partial w}{\partial z} \) if:
\[ w = r^2 + sv + r^3, \text{ with } r = x^2 + y^2 + z^2, \quad s = xyz, \quad v = xe^y, \quad t = yz^2. \]

3. [20 Points]
Solve the vector-initial value problem for \( \vec{r}(t) \) given that:
\[ \vec{r}''(t) = \vec{i} + e^t \vec{j}, \quad \vec{r}(0) = 2\vec{i}, \quad \vec{r}'(0) = 2\vec{j}. \]

4. [20 Points]
Show that the line \( l : x = -1 + t, \quad y = 3 + 2t, \quad z = -7 \) and the plane \( 2x - 2y - 2z + 4 = 0 \) are parallel, and find the distance \( D \) between them.

5. [20 Points]
(a) A Wagon is pulled along a level ground by exerting a force of 20 lb on a handle that makes an angle of 30° with the horizontal. Find the work done in pulling the Wagon 100 ft.

6. [20 Points]
Find \( \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \) if \( z = f(x, y) \) is a differentiable function determined implicitly by the equation:
\[ 2xz^3 - 3yz^2 + x^2 y^2 + 4z = 0. \]

7. [20 Points]
Find equations for the tangent plane and the normal line to the graph of the equation:
\[ F(x, y, z) = xy + 2yz - xz^2 + 70 = 0, \]
at the point \( P(-5, 5, 1) \).
8. [20 Points]
Let \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) be the function of two variables defined by:
\[
f(x, y) = 0.5x^2 + 2xy - 0.5y^2 + x - 8y.
\]
Find the local extrema of the function \( f \) or saddle point.

9. [20 Points]
Find the volume of the largest rectangular box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid:
\[
16x^2 + 4y^2 + 9z^2 = 144.
\]

10. [20 Points]
A particle \( P \) travels along a smooth curve \( C \) given by the position vector:
\[
\vec{r}(t) = \sqrt{2}t \hat{i} + t^3 \hat{j} + tk.
\]
Find the scalar and vector, tangential and normal components of the acceleration and the curvature \( \kappa(t) \) of the path \( C \) at time \( t = 1 \).
\textbf{[Hint: Scalar components of acceleration formulae are:}
\[
\vec{a}_T = \frac{\vec{v} \times \vec{a}}{\|\vec{v}\|}, \quad \vec{a}_N = \frac{\vec{a}}{\|\vec{v}\|}, \quad \kappa = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|^3}.
\]

11. [20 Points]
Evaluate the double integral \( \iint_R (2xy - x^2) dA \), where \( R \) is the rectangle bounded by \(-1 \leq x \leq 2 \) and \( 0 \leq y \leq 4 \).

12. [20 Points]
Let \( \vec{F}(x, y, z) = xy^2z^4i + (2x^2y + z)\hat{j} + y^3z^2\hat{k} \).
Find the \( \text{curl} \vec{F} = \nabla \times \vec{F} \) and the \( \text{div} \vec{F} = \nabla \cdot \vec{F} \).

13. [20 Points]
Reverse the order of integration and evaluate the resulting integral:
\[
\int_0^2 \int_0^3 ye^{-x} dx dy
\]

14. [20 Points]
Set up a surface integral to show that the unit sphere \( x^2 + y^2 + z^2 = 1 \) has surface area \( 4\pi \).
Set up a triple integral to show that the unit sphere \( x^2 + y^2 + z^2 = 1 \) has volume \( 4\pi / 3 \).

15. [20 Points]
Evaluate the line integral \( \int_C (x + 3y) dx + (x - y) dy \) along the curve \( C: x = 2\cos t, y = 6\sin t, \ 0 \leq t \leq \pi / 6 \).