Q1)
(a) Use implicit differentiation to find \( \frac{dy}{dx} \) for \( x^2 + y^3 - 2y = 3 \). Then find the equation of the tangent line to the curve at (2,1).

(b) If \( y = \int_1^x \cos t\, dt \), find \( \frac{dy}{dx} \).

(c) Find \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \).

Q2) Find the following antiderivatives:

(i) \( \int x\sqrt{2-x}\, dx \)

(ii) \( \int t^2 \sin t\, dt \)

(iii) \( \int \sin^3 x \cos^4 x\, dx \)

Q3)
(a) Let \( f(x) = \frac{1}{x+1} \), for \( |x| < 1 \). Find the power series for \( f(x) \).

(b) Using the results from part (a), deduce the power series for \( \ln(1+x) \).

Q4)
(a) Define what is meant for a sequence of functions \( \{f_n\} \) to converge uniformly on an interval \([a, b]\) to a function \( f \).

(b) Show that the sequence of functions \( \{f_n\} \) where \( f_n(x) = x^n \) is uniformly convergent on \([0, k]\) where \( k < 1 \).

(c) From part (b) above, what conclusion can you deduce if the interval under consideration is \([0, 1]\).

Q5) Determine if the following series converges or diverges:

(a) \( \sum_{n=1}^{\infty} \frac{(2n)!}{n!n!} \)

(b) \( \sum_{n=1}^{\infty} \frac{1}{n^3 + 5n + 3} \)

(c) \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \)

Q6) Find the eigenvalues and corresponding eigenvectors for: \( A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \).
Q7)
(a) Let \( u \) be a vector in \( \mathbb{R}^n \) such that \( u^T u = 1 \). Show that the \((n \times n)\) matrix \( P = uu^T \) is an idempotent matrix.
(b) Let \( Q \) be an idempotent matrix. Verify that \( I - Q \) is idempotent and show that \((I - 2Q)^{-1} = (I - 2Q)\).

Q8) By reversing the order of integration, evaluate the double integral: \( \int_0^1 \int_0^1 \frac{\sin(x)}{x} \, dx \, dy \)

Q9) Find the radius of convergence and interval of convergence for the series: \( \sum_{n=0}^{\infty} \frac{n(x + 2)^n}{3^{n+1}} \)

Q10)
(a) State the Heine-Borel Theorem.
(b) State Bolzano-Weierstrass Theorem
(c) Use the definition of a limit to show that \( \lim_{n \to \infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2 \).