Instructions: This examination consists of 16 exercises worth a total of 200 pts. Answer all questions. Show your work neatly. Calculators are not allowed.

1. [10 points] Find all real solutions. If there are none, say so.
   (a) $2x + 1 = x - 3$
   (b) $\frac{x}{x - 1} = \frac{x + 2}{x - 3}$

2. [15 points] How much pure alcohol should be added to 1 liter of a 30% alcohol solution to get a 50% solution?

3. [15 points] Solve the following equations.
   (a) $x^2 - 2x - 24 = 0$
   (b) $\sqrt{x - 2} = x - 4$

4. [15 points] Solve the following inequalities, writing your answer in interval notation.
   (a) $2x + 3 < 4x - 1$
   (b) $|3x - 1| < 5$
   (c) $|4 - x| > 2$

5. [10 points] Let $A = (2, 1)$ and $B = (-6, 5)$.
   (a) Find the distance between $A$ and $B$.
   (b) Find the midpoint of the line segment between $A$ and $B$.

6. [15 points]
   (a) Write an equation for the circle with center $(3, -1)$ and radius $\sqrt{5}$
   (b) Find the center and radius of the circle with equation
   $x^2 + y^2 - 6x + 4y - 3 = 0$.

7. [10 points] Find an equation for the line through $(6, 2)$ and perpendicular to the line passing through $(1, 1)$ and $(7, 4)$. 
8. [15 points] Write the equation expressing the facts that $t$ is jointly proportional to $r$ and $s$ and inversely proportional to $u$, and that if $r = 3$, $s = 2$, and $u = 12$, then $t = 15$.

9. [10 points] Find the domain of the function $f(x) = \frac{\sqrt{4-x^2}}{x}$.

10. [10 points] Oliver has to borrow $15,000 for a year in order to pay his school debts. His bank offers to lend him the money at a simple interest rate of 8% per year.
   (a) How much money will Oliver owe in interest?
   (b) How much does Oliver have to pay the bank at the end of the one year?

11. [10 points] Jasmine deposits $2,000 in a savings account at the bank. The bank offers 5% interest compounded quarterly. How much will Jasmine have in the savings account after 1 year? Leave your answer as a fraction raised to a power. Do not calculate.

12. [10 points] Find the average rate of change of the function $f(x) = x^4 - 4x$ between $x = -1$ and $x = 3$.

13. [10 points] How many complete revolutions a circular disk with radius 3 feet would have made when it has rolled 94.2 feet? (Take $\pi = 3.14$)

14. [15 points] Let $f(x) = 3^x$.
   a) What is $f(4)$?
   b) If $f(x) = \frac{1}{9}$, what is $x$?
   c) Sketch the graph of the function $f$.

15. [15 points] Sketch the graph of the quadratic function $f(x) = x^2 - 2x - 2$, clearly showing the $x$- and $y$- intercepts, as well as the vertex.

16. [10 points]
   a) If $4^{-x} = 7$, what is $4^{2x}$ equal to?
   b) If $2^x = 3$, what is $4^{-x}$ equal to?